CE-0732-2103
Design of Concrete Structures-I

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Design of Concrete Structures-I

COURSE CODE: CE 0732-2103 | **CREDIT:03**

TOTAL MARKS:150

Mid Exam Duration: 2 hours | **CIE MARKS: 90**

Semester End Exam Duration: 3 hours | **SEE MARKS: 60**

Course Learning Outcomes (CLOs): After completing this course successfully, the students will be able to-

CLO 1	**Understand** the fundamental behavior and properties of reinforced concrete, including the role of reinforcement in structural elements.
CLO 2	**Perform** flexural design and analysis of beams, including singly and doubly reinforced beams, using USD and WSD methods.
CLO 3	**Design and analyze** one-way slabs, stirrups, and T-beams, ensuring structural safety and code compliance.
CLO 4	**Apply** theoretical concepts to practical problems in reinforced concrete design, developing solutions for real-world engineering challenges.

SL	Content of Course	Hrs	CLOs
1	Fundamental behavior of reinforced concrete members, Properties of concrete and steel, Introduction to USD and WSD Design.	4	CLO1
2	Flexure design of Beam (Uncracked and Cracked Section, Introduction to singly-doubly beam and their USD and WSD design and analysis.)	16	CLO3
3	Differences between one way and two-way slab, Design of One-way Slab in USD and WSD Method. Shear Design of reinforced concrete beam and stirrup design. Design of T beam	10	CLO2, CLO4

Text Book:

1. Design of Concrete Structures by Arthur H. Nilson, David Darwin, Charles W. Dolan (Mc Graw Hill) – 13th edition.
2. Design of Concrete Structures by Arthur H. Nilson – 7th edition.
3. Design of Reinforced Concrete by Jack C. McCormac, Russell H. Brown – 9th edition
4. Design of Prestressed Concrete Structures by T. Y. Lin and Ned H. Burns

Week	Topic	Teaching Learning Strategy	Assessment Strategy	CLOs	Page No.
01-02	Fundamental Behavior of Reinforced Concrete Behavior	Lecture, Discussion	Mid, Final	CLO1	
03	Analysis of Singly Beam in WSD	Lecture, Discussion	Mid, Final	CLO3	
04	Analysis of Singly Beam in USD	Lecture, Discussion	Assignment, Mid, Final	CLO3	
05	Design of Singly Beam in WSD	Lecture, Discussion	Assignment, Mid, Final	CLO3	
06	Design of Singly Beam in USD	Lecture, Discussion	Assignment, Class Test, Mid, Final	CLO3	
07	Design of Doubly Beam in WSD	Lecture, Discussion	Problem solving, Class Test, Final	CLO3	
08	Design of Doubly Beam in USD	Lecture, Discussion	Final, Class Test	CLO3	
09	Analysis of Doubly Beam in USD	Lecture, Discussion	Problem solving, Class Test, Final	CLO3	
10	Analysis of Doubly Beam in WSD	Lecture, Discussion	Assignment, Final	CLO3	
11-12	Design of One-Way Slab	Lecture, Discussion	Assignment, Final	CLO2, CLO4	
13	Shear Design Procedure according to ACI	Lecture, Discussion	Problem solving, Class Test, Final	CLO2, CLO4	
14	Design of Stirrup	Lecture, Discussion	Final, Class Test	CLO2, CLO4	
15	Design of T-Beam	Lecture, Discussion	Assignment, Final	CLO2, CLO4	

ASSESSMENT PATTERN

CIE- Continuous Internal Evaluation (90 Marks)

Bloom's Category Marks (out of 90)	Tests (45)	Assignments (15)	Quizzes (15)	External Participation in Curricular/Co-Curricular Activities (15)
Remember	10		10	Attendance 15
Understand	5		05	
Apply	10			
Analyze	10			
Evaluate	5			
Create	5	15		

SEE- Semester End Examination (60 Marks)

Bloom's Category	Tests
Remember	**10**
Understand	**10**
Apply	**10**
Analyze	**15**
Evaluate	**10**
Create	**5**

Fundamentals of Surveying

(Week 1)

UNIVERSITY OF GLOBAL VILLAGE

Fundamental Behavior of Reinforced Concrete Members

(Week 01-02)

Concrete

Concrete is a stonelike material obtained by permitting a carefully proportioned mixture of **cement**, **sand** and **gravel** or **other aggregates**, and **water** to harden in forms of the shape and dimensions of the desired structure.

Advantages of Concrete

- High compressive strength
- Free from corrosion
- No appreciable effects of atmospheric agents
- Not burnt in normal temperature
- Any desirable shape can be made
- Low cost
- Durable and sustainable

Disadvantages of Concrete

- Less tensile strength
- Difficult to control the quality
- It can’t be used until hardening

Factors Affecting The Strength of Concrete

- Water-cement ratio
- Size of aggregate
- Proportion of aggregate
- Curing
- Concrete porosity
- Degree of compaction

Reinforced Concrete

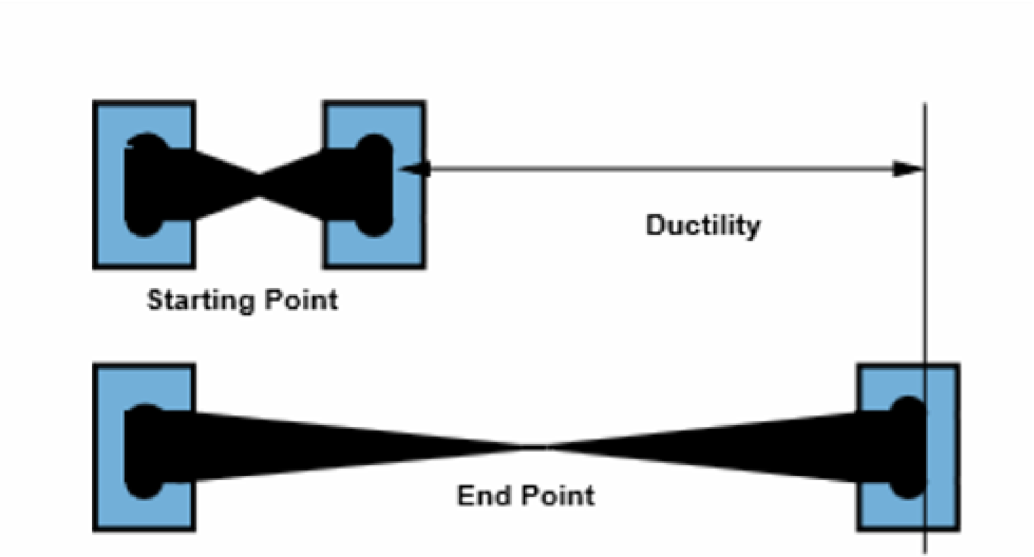
The reinforcement, usually round steel rods with appropriate surface deformations to provide interlocking, is placed in the forms in advance of the concrete. When completely surrounded by the hardened concrete mass, it forms an integral part of the member. The resulting combination of two materials, known as reinforced concrete.

Advantages of Reinforced Concrete Over Plain Concrete

- Economical for same strength
- High tensile strength
- More durable
- Provide ductility and toughness
- Good compressive strength

Ductility

Ductility is a physical property of a material that describes its ability to be stretched, pulled, or drawn into a thin wire or thread without breaking. It is the measure of how much a material can be deformed or elongated under stress before it fractures.



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Why Mild Steel is used as Reinforcement?

- High tensile strength
- Develop perfect bond with concrete
- Coefficients for thermal expansion are nearly same
- Easily available
- Can be cut and bent easily
- Produce no harmful effect combining with concrete

Load

Dead load
Constant in magnitude, fixed in location.
Example: Self weight of structure, floor finish, floor fill and plastered ceiling

Live Load
Magnitude and location is uncertain.
Example: Occupancy loads

Environmental Load
Example: Snow loads, wind pressure, earthquake loads, soil pressures, loads from ponding of rainwater, and forces caused by temperature differentials.

Minimum Uniformly Distributed Live Loads

Occupancy or Use	Live Load (psf)
Residential	40
Classroom	40
Office use	50
Computer use	100
Staircase	100
Balcony	100
Storage Warehouse (light)	125
Storage Warehouse (heavy)	150

Reference: Table 6.2.3, BNBC 2020

Sources of Uncertainities

- Actual loads may differ from those assumed.
- Actual loads may be distributed in a manner different from that assumed.
- The assumptions and simplifications inherent in any analysis may result in calculated load effects- moments, shears, etc.
- The actual structural behavior may differ from that assumed, owing to imperfect knowledge.
- Actual member dimensions may differ from those specified.
- Reinforcement may not be in its proper position.
- Actual material strength may be different from that specified.

Safety Provisions of ACI Code

$$\boldsymbol{M_u = \Phi M_n}$$

$$\Phi = Strength\ Reduction\ Factor$$

$$Factored\ Load = 1.2\ DL + 1.6\ LL$$

$$Service\ Load = DL + LL$$

Strength Reduction Factor

To provide structural safety, the ultimate strength is reduced by a coefficient which is known as strength reduction factor. It is denoted by Φ.

The value for bending is higher than that for shear or bearing. Because the nature of flexure failure is ductile and depends on the quality of steel. And the nature of shear failure is brittle and depends on the quality of concrete, which varies from time to time.

Strength Reduction Factors in the ACI Code

Strength Condition	Strength Reduction Factors, Φ
Tension-controlled sections	0.90
Compression-controlled section	
Members with Spiral Reinforcement	0.70
Other Reinforced Members	0.65
Shear and torsion	0.75
Bearing on concrete	0.65
Post-tensioned anchorage zones	0.85
Strut-and-tie models	0.75

FUNDAMENTAL ASSUMPTIONS FOR REINFORCED CONCRETE BEHAVIOR

- The internal forces, such as bending moments, shear forces, and normal and shear stresses, at any section of a member are in equilibrium with the effects of the external loads at that section.

- The strain in an embedded reinforcing bar is the same as that of the surrounding concrete.

- Cross sections that were plane prior to loading continue to be plane in the member under load.

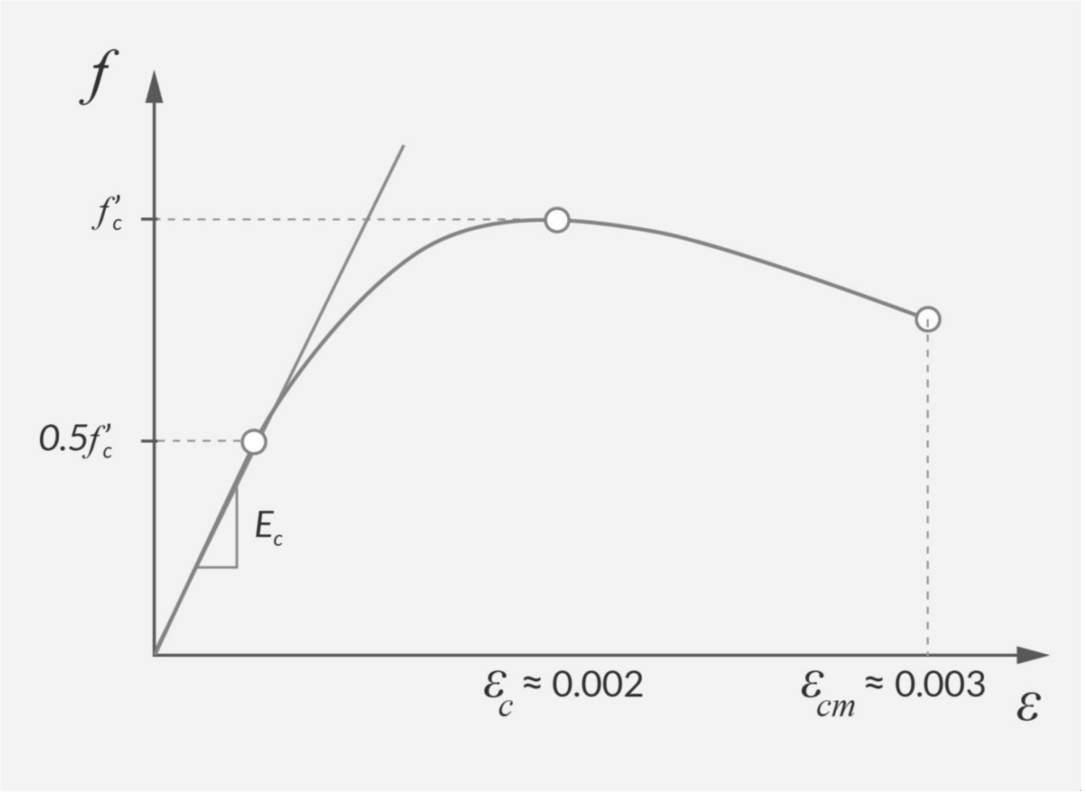
- concrete is not capable of resisting any tension stress.

- The theory is based on the actual stress-strain relationships and strength properties of the two constituent materials or some reasonable equivalent simplifications thereof.

Elastic Behavior of Concrete

At low stresses, up to about $f'_c/2$, the concrete is seen to behave nearly elastically, i.e., stresses and strains are quite closely proportional.

$$\sigma \propto \varepsilon$$



From Assumption 2,

$$\varepsilon_c = \varepsilon_s$$

$$\frac{f_c}{E_c} = \frac{f_s}{E_s}$$

$$f_s = \frac{E_s}{E_c} \times f_c$$

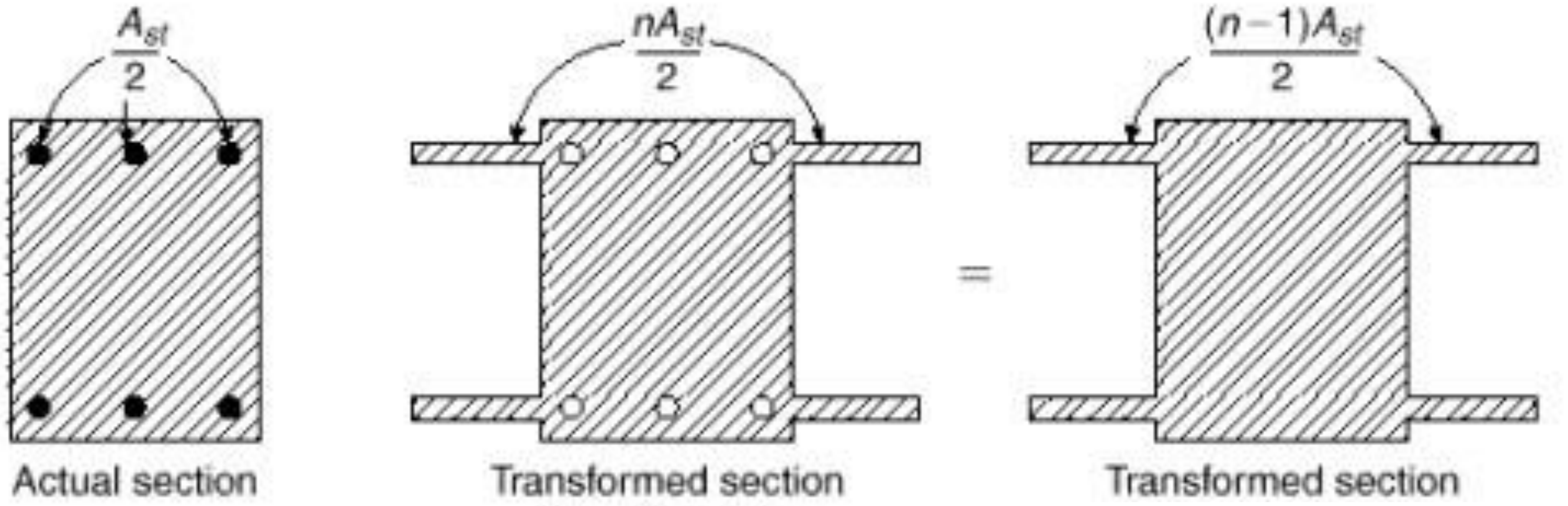
$$f_s = n \times f_c$$

Modular ratio, $n = \frac{E_s}{E_c}$

Reinforcing Bars

Bar No.	Bar Dia (in) $[\frac{Bar\ No.}{8}]$	Area (in^2) $[\frac{\pi}{4} \times dia^2]$
3	3/8	.11
4	4/8	.20
5	5/8	.31
6	6/8	.44
7	7/8	.60
8	1	.79
9	9/8	1
10	10/8	1.27

Transformed Section



$A_t = A_c + nA_{st}$

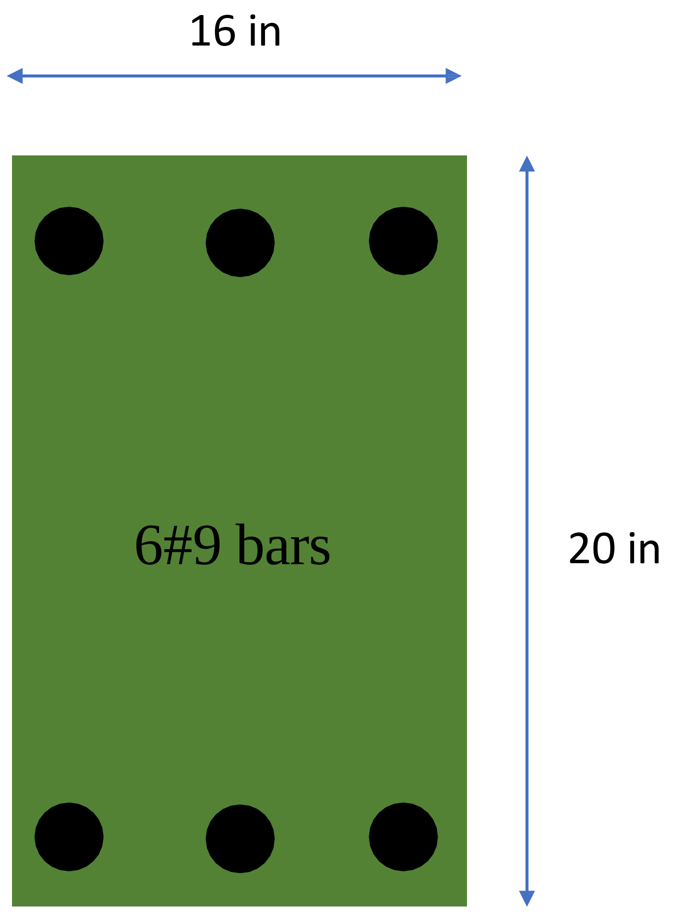
$A_t = A_g + (n-1)A_{st}$

A_t : Transformed area
A_c : Net area of concrete
A_g : Gross area
A_{st} : Area of steel
n : Modular ratio

$$Axial\ load, P = P_c + P_s$$
$$= f_c\, A_c + f_s\, A_{st}$$
$$= f_c\, A_c + nf_c\, A_{st}$$
$$= f_c\, \mathbf{(A_c + n\, A_{st})}$$
$$= f_c\, (A_g - A_{st} + \text{n}\, A_{st})$$
$$= f_c\,[\boldsymbol{A_g} + (\boldsymbol{n} - \mathbf{1})\boldsymbol{A_{st}}]$$

Problem

A column has a cross-section of 16in x 20in and reinforced by six No. 9 bars. Determine the axial load that will stress concrete to 1200 psi.



Solution

Gross area, $A_g = 16 \times 20 = 320\ in^2$

Area of Steel, $A_{st} = 6 \times 1 = 6\ in^2$

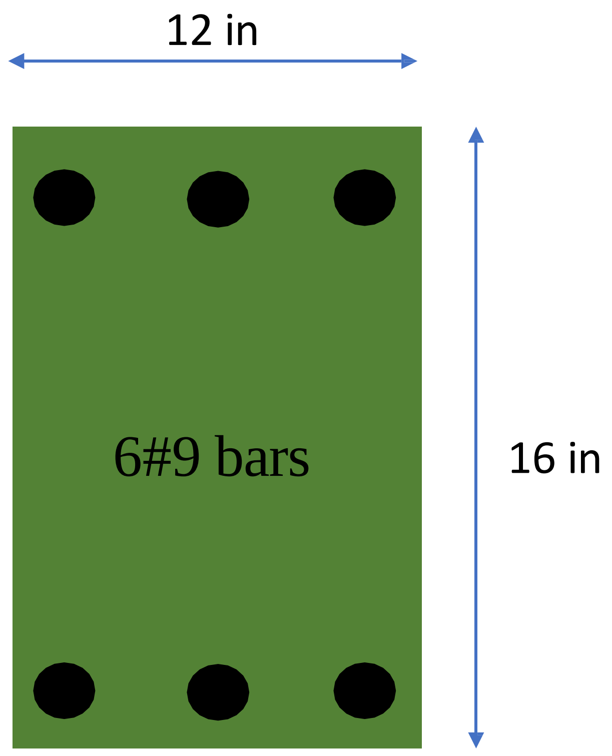
Modular ratio, $n = \frac{E_s}{E_c} = 8\ \ [Let]$

P = ?

$$P = f_c[\boldsymbol{A_g} + (\boldsymbol{n} - \mathbf{1})\boldsymbol{A_{st}}]$$

$$= 1200 \times [320 + (8 - 1) \times 6]$$

$=$ **434400 lb** (Ans)



Solution

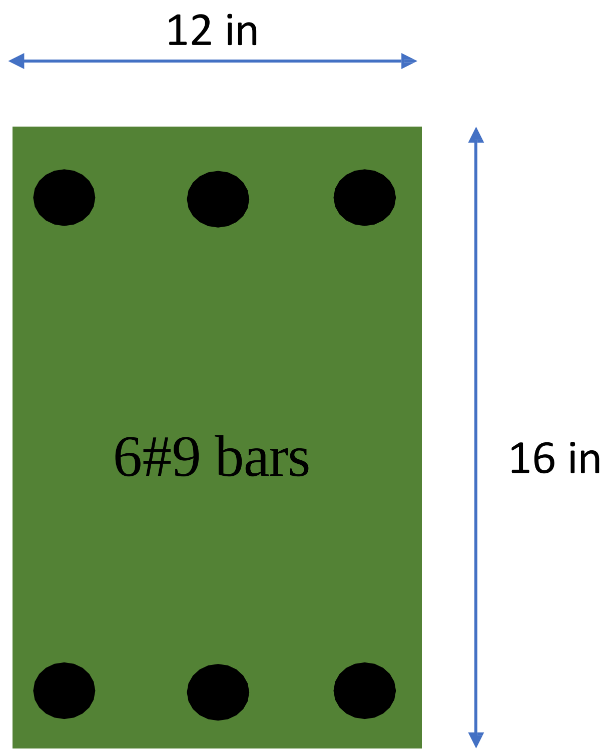
Load carried by concrete, $P_c = f_c A_c = 1200 \times (320 - 6)$

$$= 376800\ lb$$

Load carried by steel, $P_s = f_s A_{st} = n f_c A_{st}$

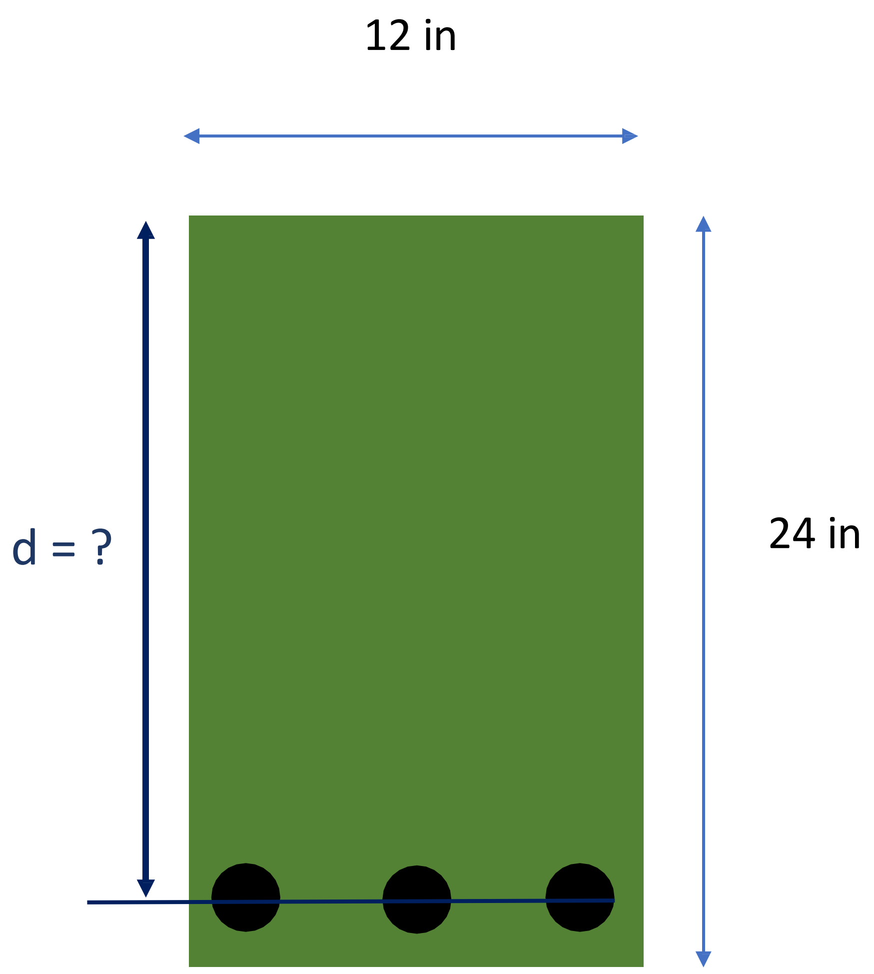
$$= 8 \times 120 \times 6$$

$$= 57600\ lb$$



Effective Depth

Distance from the centroid of the reinforcement to the compression face of the section is known as effective depth.



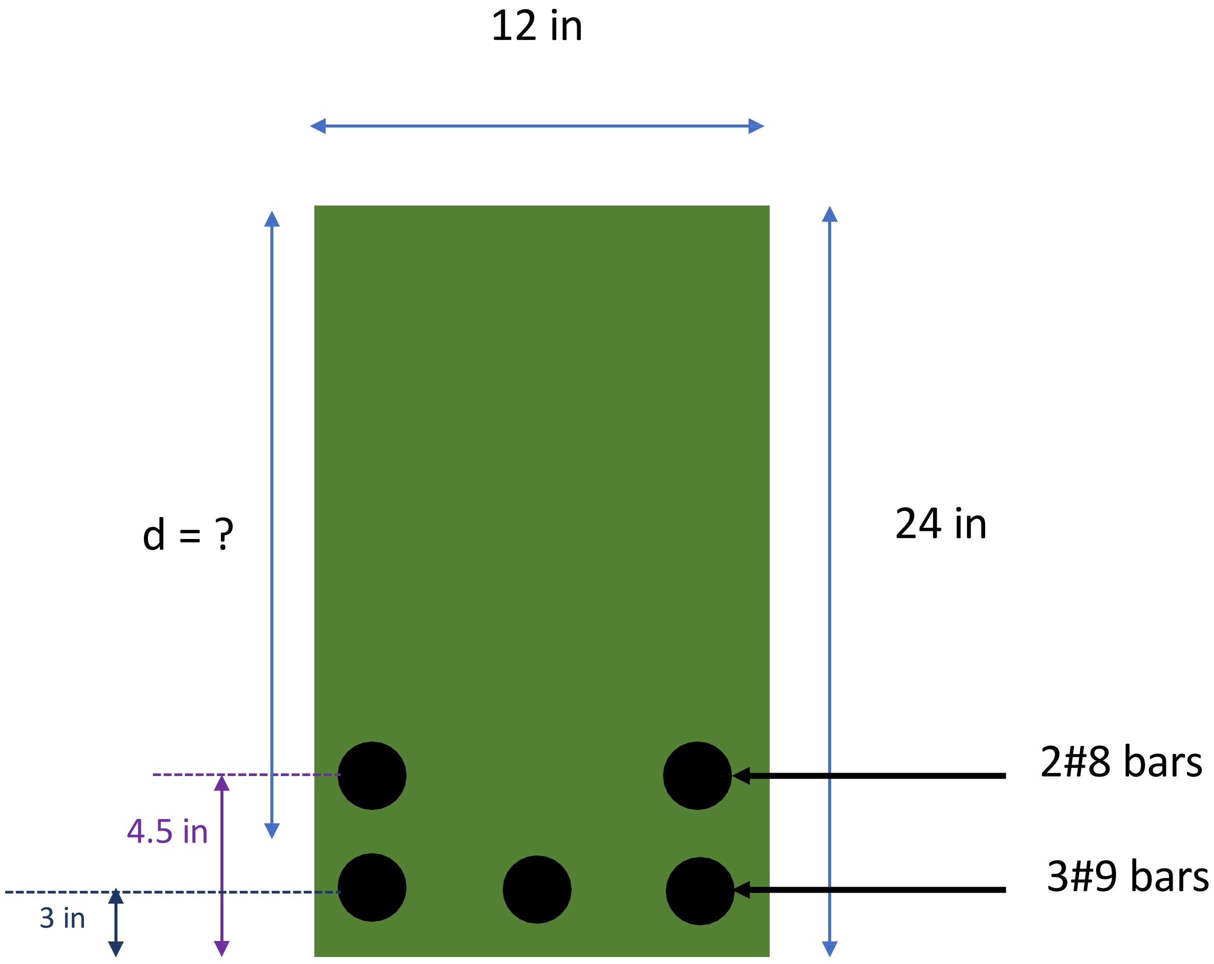
Effective Depth

$$y = \frac{A_1 y_1 + A_2\, y_2}{A_1 + A_2}$$

$$= \frac{(3\times3)+(1.58\times4.5)}{(3+1.58)}$$

$$= 3.51\ in$$

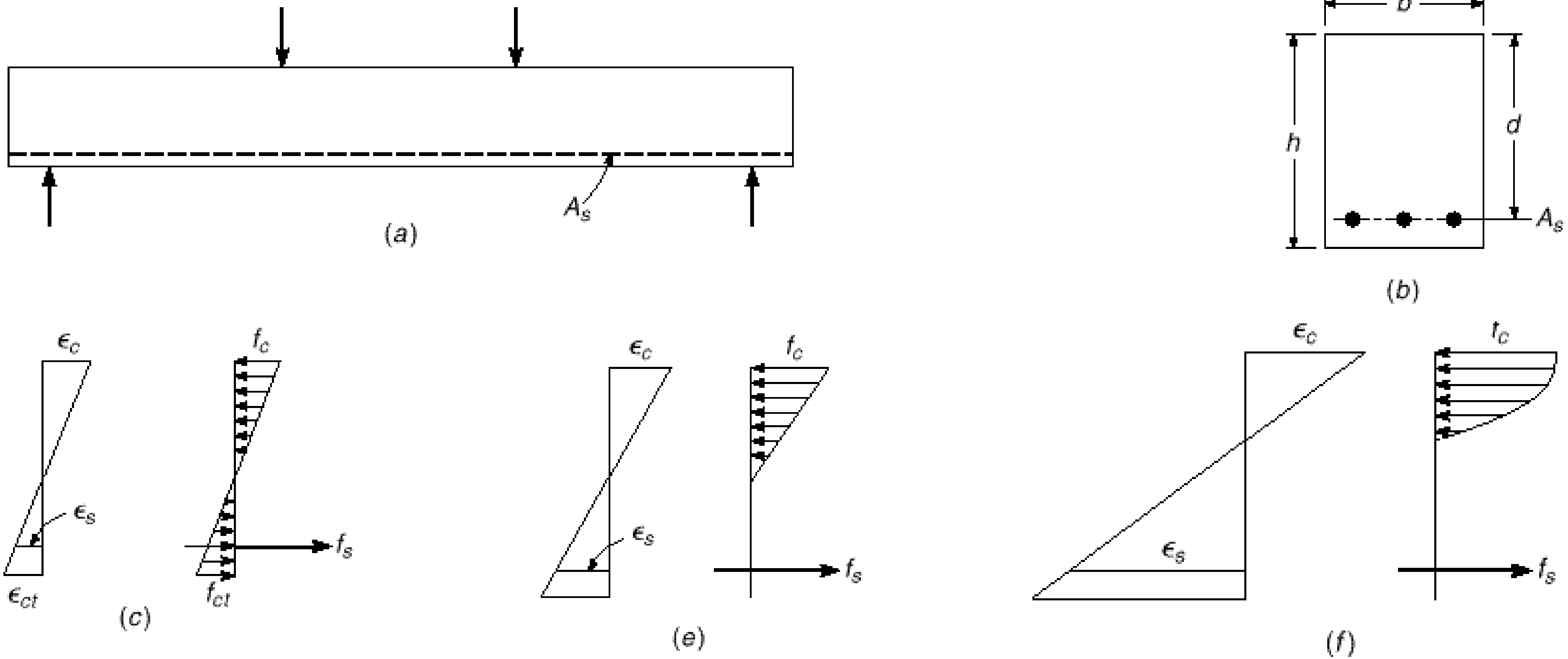
$$Effective\ depth, d = t - y$$
$$= 24 - 3.51$$
$$= \mathbf{20.49\ in}$$



Behavior of Reinforced Concrete Beam

- Stress elastic and section uncracked (No cracking)
- Stress elastic and section cracked (Upto elastic limit)
- Ultimate flexural strength (Just before failure)

Behavior of Reinforced Concrete Beam



Stress elastic and section uncracked

Stress elastic and section cracked

Ultimate flexural strength

A rectangular beam has the dimensions b = 10 in, h = 25 in and d = 23 in, and is reinforced with three No. 8 bars so that A_s = 2.37 in^2. The concrete cylinder strength, f'_c = 4000 psi and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y = 60,000 psi. Determine the stresses caused by a bending moment M = **45 ft-kips**.

Solution:

Here,

b : 10 in

h : 25 in

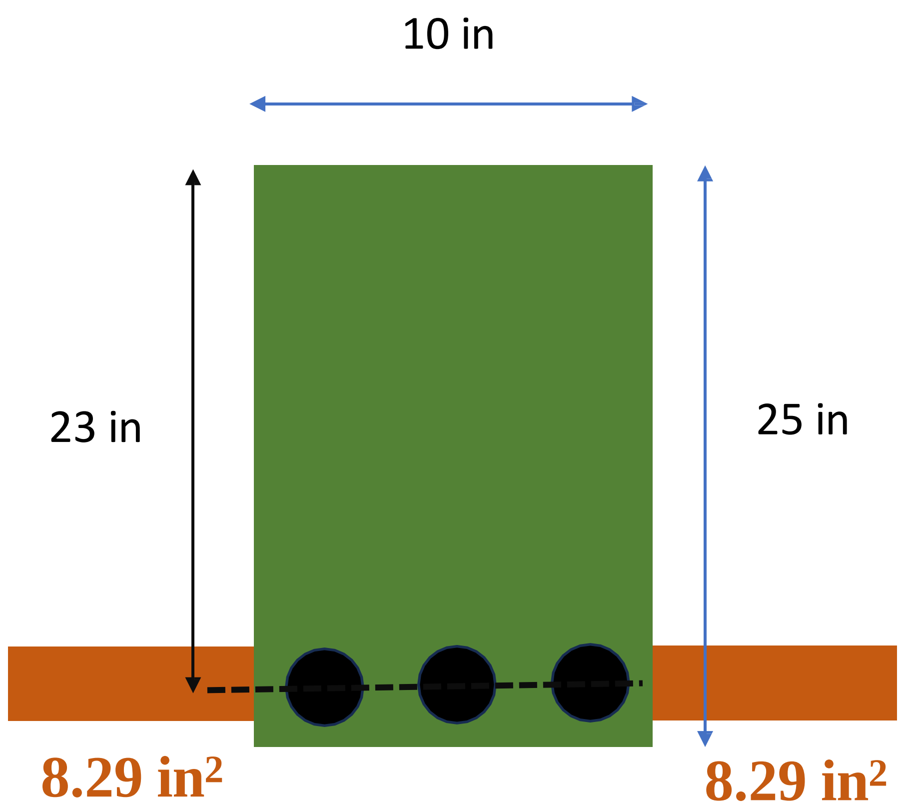
d : 23 in

M : 45 ft-kips

A_s : (3 x .79) = 2.37 in^2

n : E_s/E_c = 8

(n-1) A_s: (7 x 2.37) = **16.59 in^2**



A rectangular beam has the dimensions b = 10 in, h = 25 in and d = 23 in, and is reinforced with three No. 8 bars so that A_s = 2.37 in². The concrete cylinder strength, f'_c = 4000 psi and the tensile strength of concrete in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y = 60,000 psi. Determine the stresses caused by a bending moment M = **45 ft-kips**.

Solution:

Location of Neutral Axis from top, $\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(10\times25\times12.5)+(16.59\times23)}{[(10\times25)+16.59]} = 13.2\ in$

Moment of your Inertia about the N.A.

$= \left(\frac{bh^3}{12} + Ad^2\right) + (Ad^2)$

$= [\frac{10\times25^3}{12} + (10 \times 25 \times (13.2 - 12.5)^2] + [16.59 \times (23 - 13.2)^2]$

= 14736.63 in⁴

A rectangular beam has the dimensions b = 10 in, h = 25 in and d = 23 in, and is reinforced with three No. 8 bars so that $A_s = 2.37\ in^2$. The concrete cylinder strength, $f'_c = 4000$ psi and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel $f_y = 60{,}000$ psi. Determine the stresses caused by a bending moment M = **45 ft-kips**.

Solution:

Tensile stress of concrete, $f_{ct} = \frac{M \times c}{I}$

$$= \frac{(45 \times 12 \times 1000) \times (25 - 13.2)}{14736.63} = 432 \text{ psi} < 475 \text{ psi}$$

No crack will be formed.

Compressive stress of Concrete, $f_c = \frac{Mc}{I}$

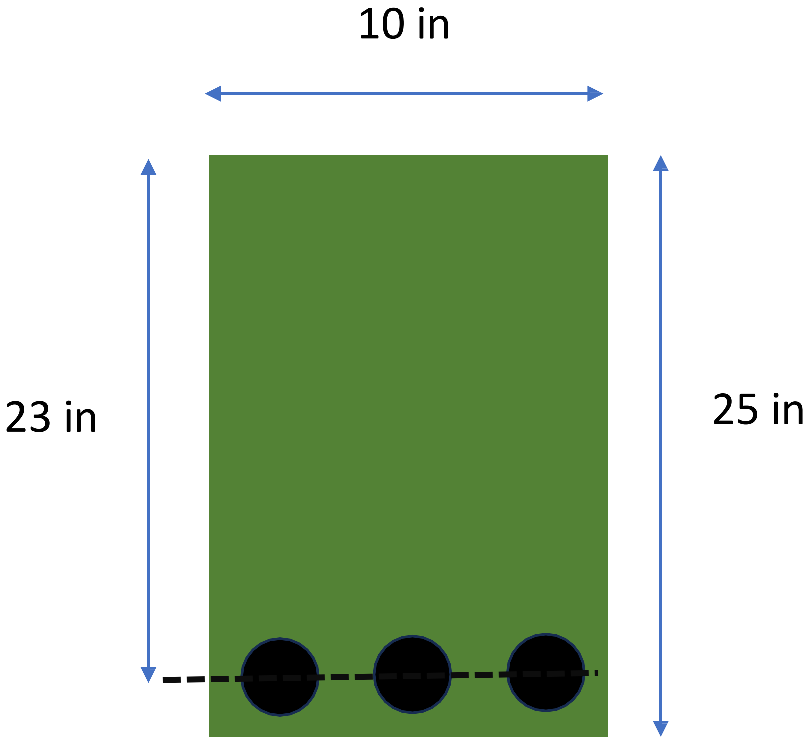
$$= \frac{(45 \times 12 \times 1000) \times 13.2}{14736.63} = \mathbf{484\ psi}\ \text{(Ans)}$$

Compressive stress of Steel, $f_s = nf_c = n\frac{Mc}{I}$

$$= 8 \text{ x } \frac{(45 \times 12 \times 1000) \times 9.8}{14736.63} = \mathbf{2870\ psi}\ \text{(Ans)}$$

A rectangular beam has the dimensions b = 10 in, h = 25 in and d = 23 in, and is reinforced with three No. 8 bars so that $A_s = 2.37\ in^2$. The concrete cylinder strength, f'_c = 4000 psi and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y = 60,000 psi. Determine the stresses caused by a bending moment M = **90 ft-kips**.

Solution:



Here,

b : 10 in

h : 25 in

d : 23 in

M : 90 ft-kips

A_s : (3 x .79) = 2.37 in^2

A rectangular beam has the dimensions b = 10 in, h = 25 in and d = 23 in, and is reinforced with three No. 8 bars so that A_s = 2.37 in^2. The concrete cylinder strength, f'_c = 4000 psi and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y = 60,000 psi. Determine the stresses caused by a bending moment M = **90 ft-kips**.

Solution:

Concrete tension stress at bottom fiber,

$$f_{ct} = \frac{M \times c}{I} = \frac{(90\times12\times1000)\times11.8}{14740} = 864 \text{ psi} > 475 \text{ psi}$$

Section will be cracked.

Reinforcement ratio, $\rho = \frac{A_s}{bd} = \frac{2.37}{10\times23} = 0.0103$

$$n = \frac{E_s}{E_c} = \frac{29000000}{57000\sqrt{4000}} \approx 8$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n = 0.332$$

$$j = 1 - \frac{k}{3} = 0.889$$

Location of Neutral Axis, kd = 7.63 in

Steel Stress, $f_s = \frac{M}{A_s\, j\, d} = \frac{(90\times12\times1000)}{2.37\times.889\times23} =$ **22287 psi** (Ans)

Concrete Stress, $f_c = \frac{2M}{kjbd^2} = \frac{2\times(90\times12\times1000)}{.332\times.889\times10\times23^2} =$ **1350 psi** (Ans)

Under–reinforced Section

When $\rho < \rho_b$, the section is considered as under–reinforced Section.
In this case, tension failure will occur.

Overreinforced Section

When $\rho > \rho_b$, the section is considered as over–reinforced Section.
In this case, compression failure will occur.

A rectangular beam has the dimensions b = 10 in, h = 25 in and d = 23 in, and is reinforced with three No. 8 bars so that A_s = 2.37 in^2. The concrete cylinder strength, f'_c = 4000 psi and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y = 60,000 psi. Determine the nominal moment at which the beam will fail.

Solution:

Checking if the section is under-reinforced or over-reinforced

$$\rho = \frac{A_s}{bd} = \frac{2.37}{10 \times 23} = 0.0103$$

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right) = 0.85 \times 0.85 \times \frac{4000}{60000}\left(\frac{0.003}{0.003 + \frac{60000}{29000000}}\right) = 0.0335$$

Since $\rho < \rho_b$, the section is under-reinforced and tension failure is expected to occur.

$$\therefore \boldsymbol{f_s = f_y}$$

A rectangular beam has the dimensions b = 10 in, h = 25 in and d = 23 in, and is reinforced with three No. 8 bars so that A_s = 2.37 in^2. The concrete cylinder strength, f'_c = 4000 psi and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y = 60,000 psi. Determine the nominal moment at which the beam will fail. Also determine the ultimate moment.

Solution: [According to General Analysis]

$$M_n = \rho f_y \mathrm{b} d^2\left[1-\left(0.59\rho \times \frac{f_y}{f'_c}\right)\right]$$

$$M_n = 0.0103 \times 60000 \times 10 \times 23^2\left[1-\left(\frac{0.59 \times 0.0103 \times 60000}{4000}\right)\right]$$

$M_n = 2970000\ in - lb = \mathbf{248\ ft - kips}$ (Ans)

Ultimate moment, $M_u = \Phi M_n = 0.90 \times 248 = \mathbf{223\ ft - kips}$ (Ans)

A rectangular beam has the dimensions b = 10 in, h = 25 in and d = 23 in, and is reinforced with three No. 8 bars so that A_s = 2.37 in^2. The concrete cylinder strength, f'_c = 4000 psi and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y = 60,000 psi. Determine the nominal moment at which the beam will fail. Also determine the ultimate moment.

Solution: [According to Equivalent Rectangular Stress Distribution]

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.37 \times 60000}{0.85 \times 4000 \times 10} = 4.18\ in$$

$$M_n = \boldsymbol{A_s f_y}(\mathbf{d}\ - \boldsymbol{\frac{a}{2}})$$

$$M_n = 2.37 \times 60000 \left[23 - \frac{4.18}{2}\right]$$

$M_n = \boldsymbol{248\ ft - kips}$ (Ans)

Ultimate moment, $M_u = \Phi M_n = 0.90 \times 248 =\ \boldsymbol{223\ ft - kips}$ (Ans)

Analysis of Singly Reinforced Beam (WSD)

WEEK-03

Necessary Formulas

Permissible/Allowable Stress:

$$\boldsymbol{f_s = 0.4f_y}$$

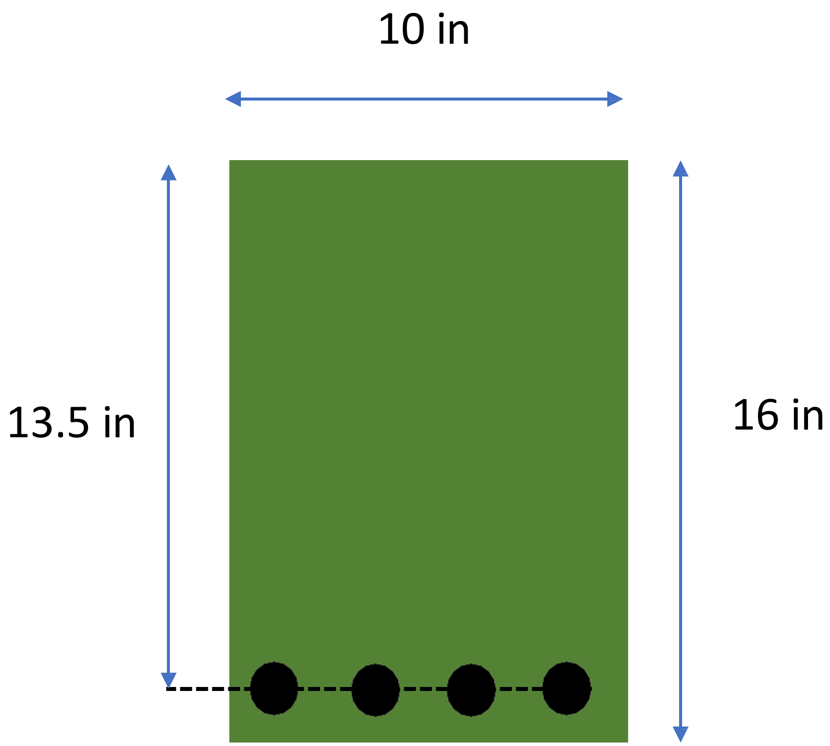
$$\boldsymbol{f_c = 0.5f'_c}$$

$Stess\ Ratio$:

$$\boldsymbol{r = \frac{f_s}{f_c}}$$

A simply supported rectangular beam has a total cross section of 10 in $\times$ 16 in and a length of 20ft. It is reinforced with 4#5 bars in one row. The distance from the center of the bars to the lower surface of the beam is 2.5 in with 2500 psi concrete and an allowable stress of 20000 psi in the steel. What is the resisting moment of the beam.

Solution:



Here,

b	: 10 in
h	: 16 in
d	: (16-2.5) = 13.5 in
L	: 20 ft
A_s	: (4 x .31) = 1.24 in^2

A simply supported rectangular beam has a total cross section of $10 \text{ in} \times 16\ in$ and a length of 20ft. It is reinforced with 4#5 bars in one row. The distance from the center of the bars to the lower surface of the beam is 2.5 in with 2500 psi concrete and an allowable stress of 20000 psi in the steel. What is the resisting moment of the beam.

Solution:

Reinforcement ratio, $\rho = \frac{A_s}{bd} = \frac{1.24}{10\times13.5} =$ 0.00919

n = $\frac{E_s}{E_c} = \frac{29000000}{57000\sqrt{2500}} = 10.175 \approx 10$

$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n = 0.347$

$j = 1 - \frac{k}{3} = 0.884$

$M_c = \frac{f_c}{2} kjbd^2 = 314464.63\ lb - in = 314$ k-in

$M_s = A_s f_s jd$ = 296 k-in

Minimum will be considered as working moment capacity.

Ans: 296 k-in

Calculate the working moment capacity of the section given below. Assume fc' = 30000 psi and f_y = 60000 psi.

Solution:

Here,

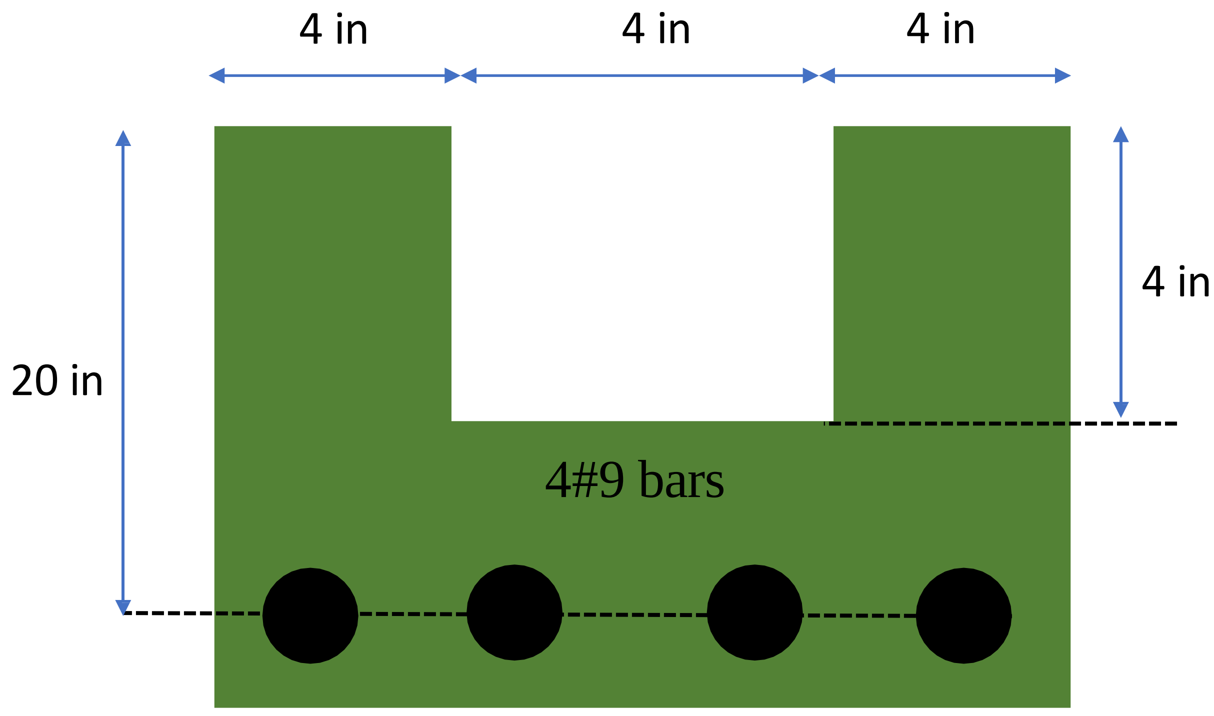
d : 20 in

A_s : (4 x 1) = 4 in^2

$$n = \frac{E_s}{E_c} = \frac{29000000}{57000\sqrt{3000}} \approx 9$$

k = ?

j = ?

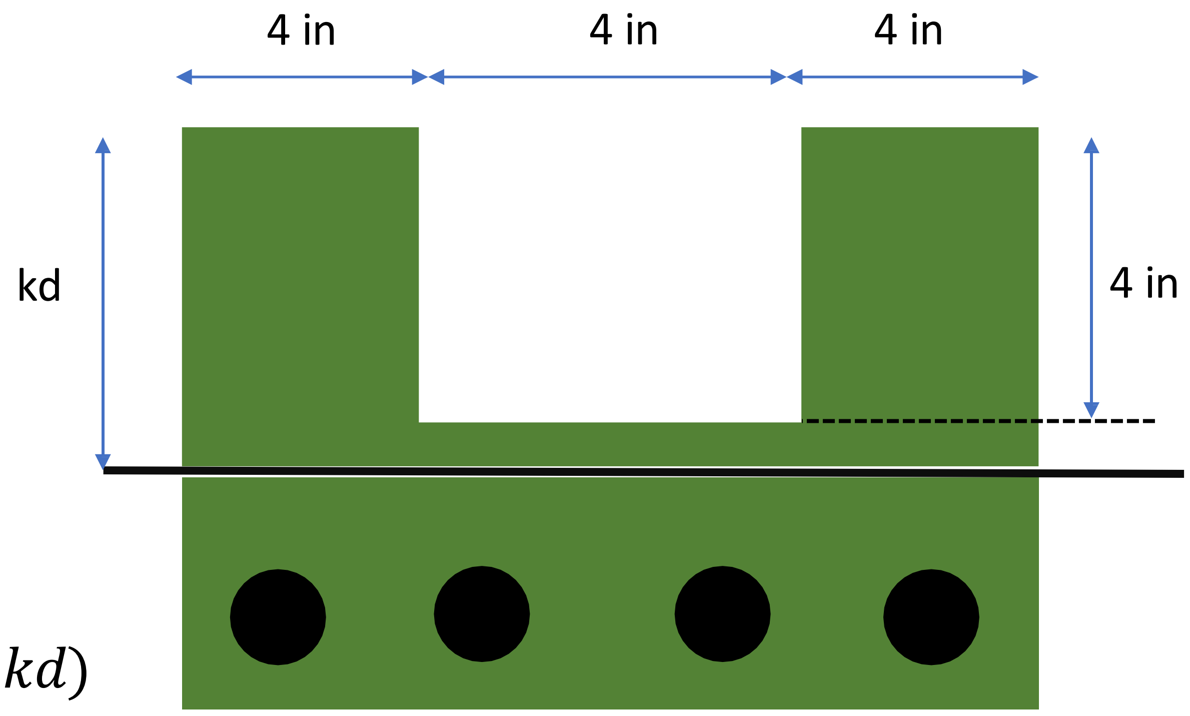


Calculate the working moment capacity of the section given below. Assume fc’ = 30000 psi and f_y = 60000 psi.

Solution:

$$Q_c = 12 \times kd \times \frac{kd}{2} - (4 \times 4)\left(kd - \frac{4}{2}\right)$$

$$Q_T = nA_{st} \times (d - kd)$$



At equilibrium,

$$Q_c = Q_T$$

$$12 \times kd \times \frac{kd}{2} - (4 \times 4)\left(kd - \frac{4}{2}\right) = nA_{st} \times (d - kd)$$

$$6 \times (20k)^2 - 16(20k - 2) = (9 \times 4) \times (20 - 20k)$$

$$\boldsymbol{k = 0.4585}$$

Calculate the working moment capacity of the section given below. Assume fc’ = 30000 psi and f_y = 60000 psi.

Solution:

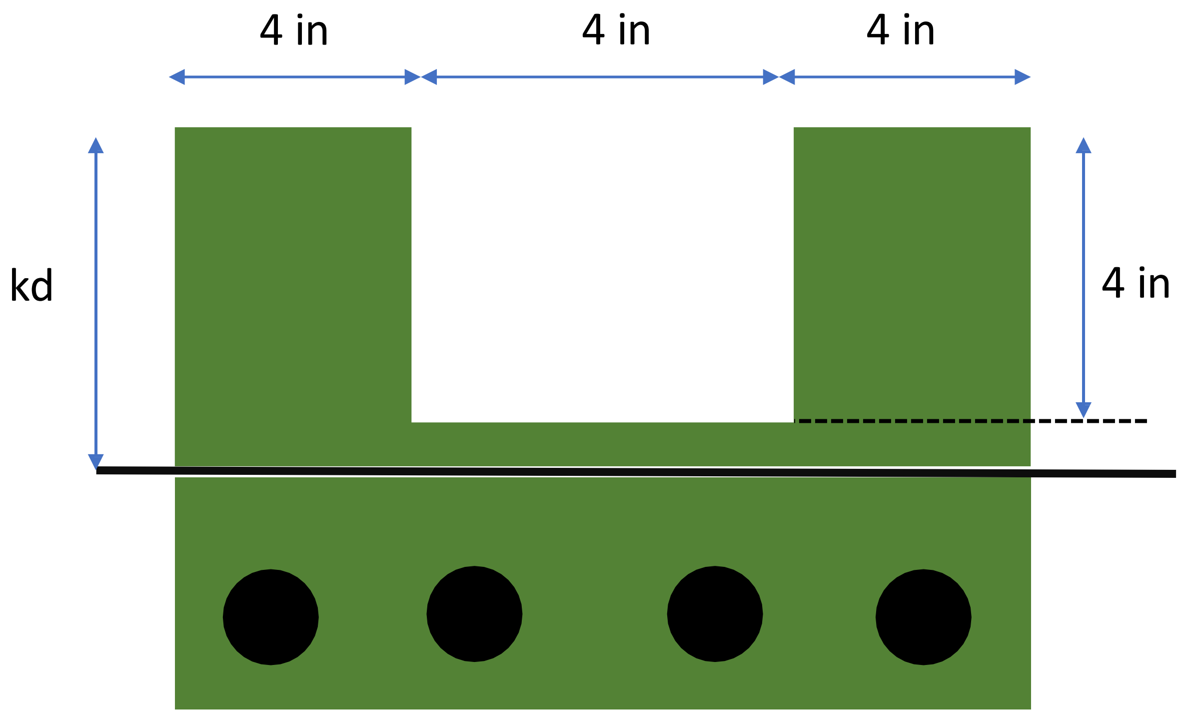
$j = 1 - \frac{k}{3}$ = 0.847

$M_c = \frac{f_c}{2} kjbd^2 = 1258252\ lb - in = 1258$ k-in

$M_s = A_s f_s jd$ = 1626240 lb-in = 1626 k-in

Minimum will be considered as working moment capacity.

Ans: 1626 k-in



Determine the moment capacity of the section given below.
Assume fc' = 3000 psi and f_y = 60000 psi.

Solution:

Here,

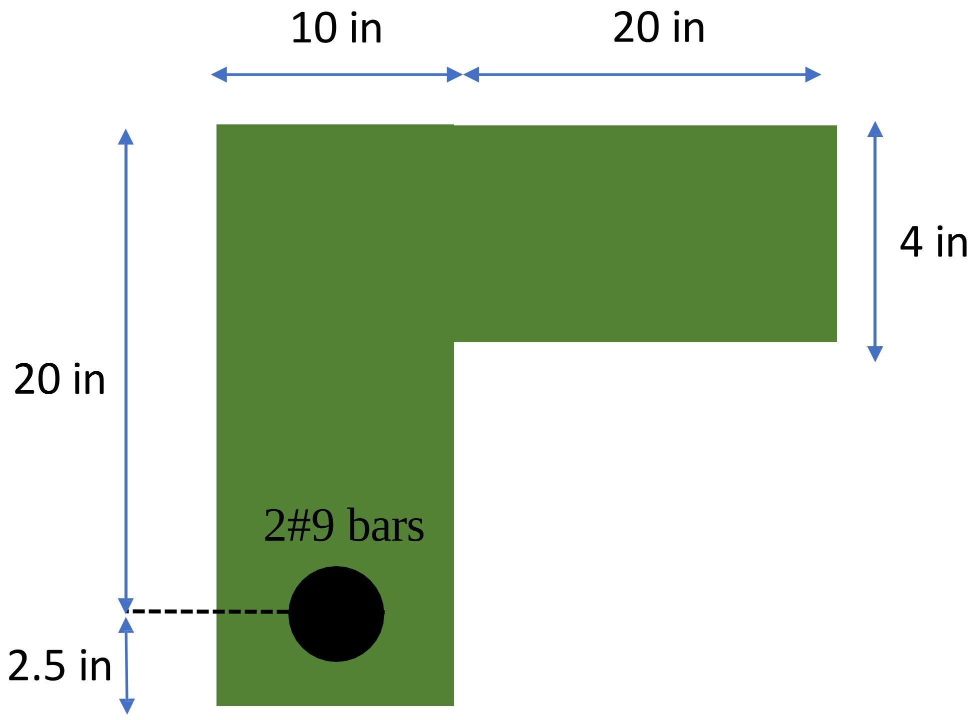
d : 20 in

A_s : (2 x 1) = 2 in²

$$n = \frac{E_s}{E_c} = \frac{29000000}{57000\sqrt{3000}} \approx 9$$

k = ?

j = ?



Calculate the working moment capacity of the section given below.
Assume fc' = 3000 psi and f_y = 60000 psi.

Solution:

$$Q_c = 10 \times kd \times \frac{kd}{2} - (20 \times 4)\left(kd - \frac{4}{2}\right)$$

$$Q_T = nA_{st} \times (d - kd)$$

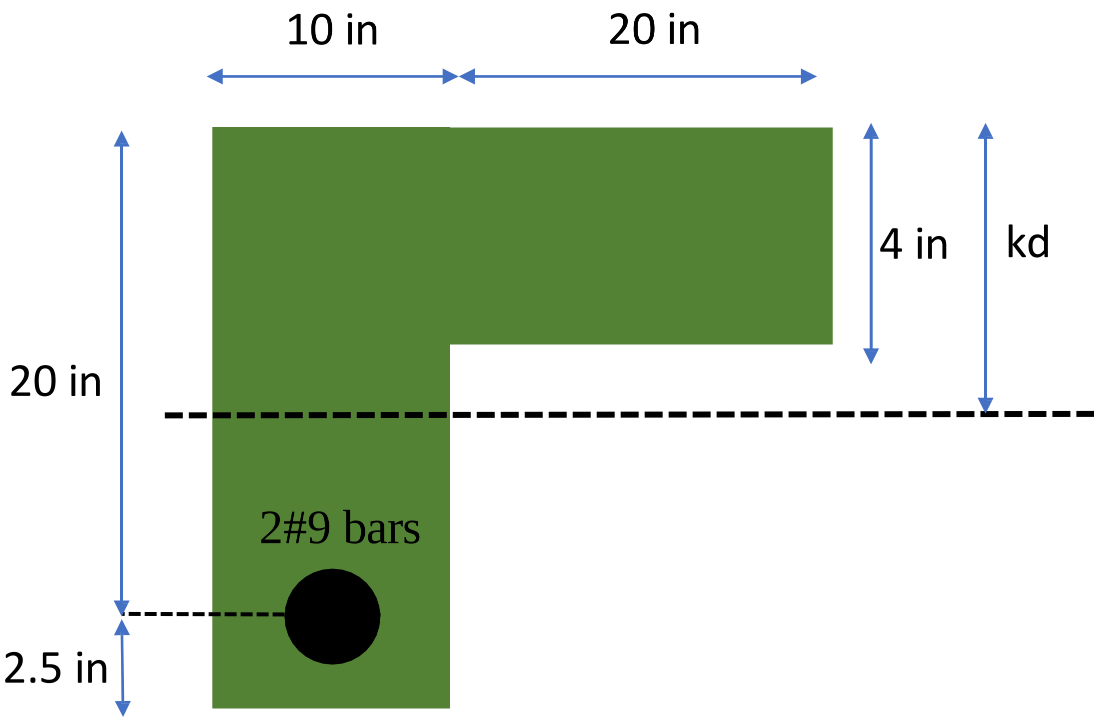
At equilibrium,

$$Q_c = Q_T$$

$$10 \times kd \times \frac{kd}{2} - (20 \times 4)\left(kd - \frac{4}{2}\right) = nA_{st} \times (d - kd)$$

$$6 \times (20k)^2 - 80(20k - 2) = (9 \times 2) \times (20 - 20k)$$

$$\boldsymbol{k = 0.217}$$



Calculate the working moment capacity of the section given below. Assume fc' = 3000 psi and f_y = 60000 psi.

Solution:

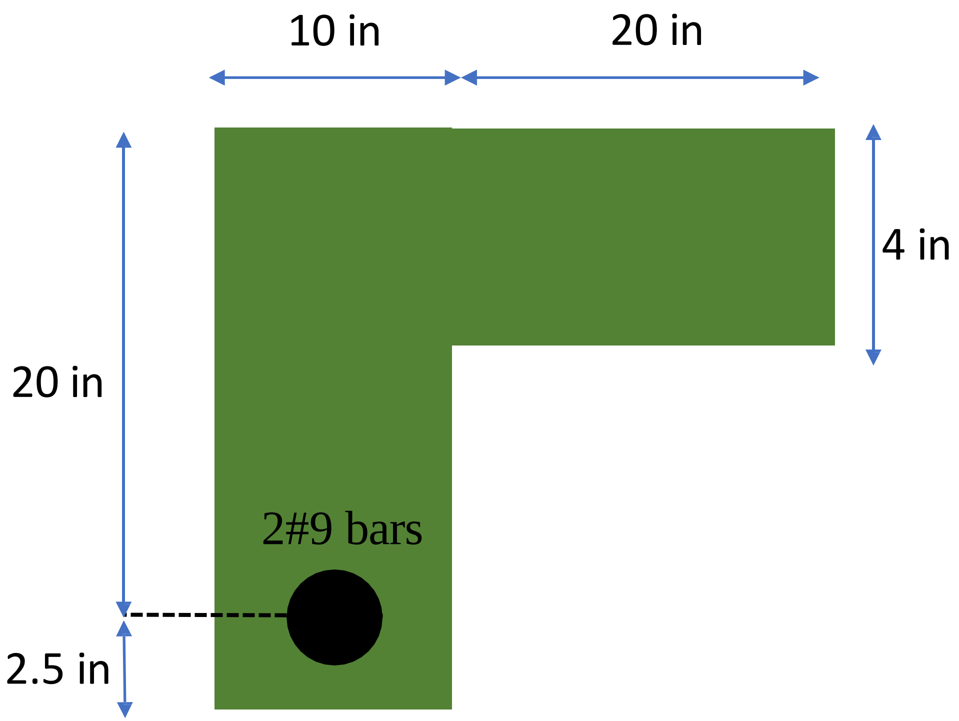
$j = 1 - \frac{k}{3}$ = 0.928

$M_c = \frac{f_c}{2} kjbd^2 = 543715\ lb - in = 45.3$ k-ft

$M_s = A_s f_s jd$ = 890880 lb-in = 74 k-ft

Minimum will be considered as working moment capacity.

Ans: 45.3 k-ft



Analysis of Singly Reinforced Beam (USD)

WEEK-04

Analysis 01: A simply supported rectangular beam has a total cross section of 12 in $\times$ 20 in. It is reinforced with 4#9 bars in one row. What is the nominal flexural strength and what is the maximum moment that can be utilized in design.
Assume fc' = 4000 psi and f_y = 60000 psi.

Solution:

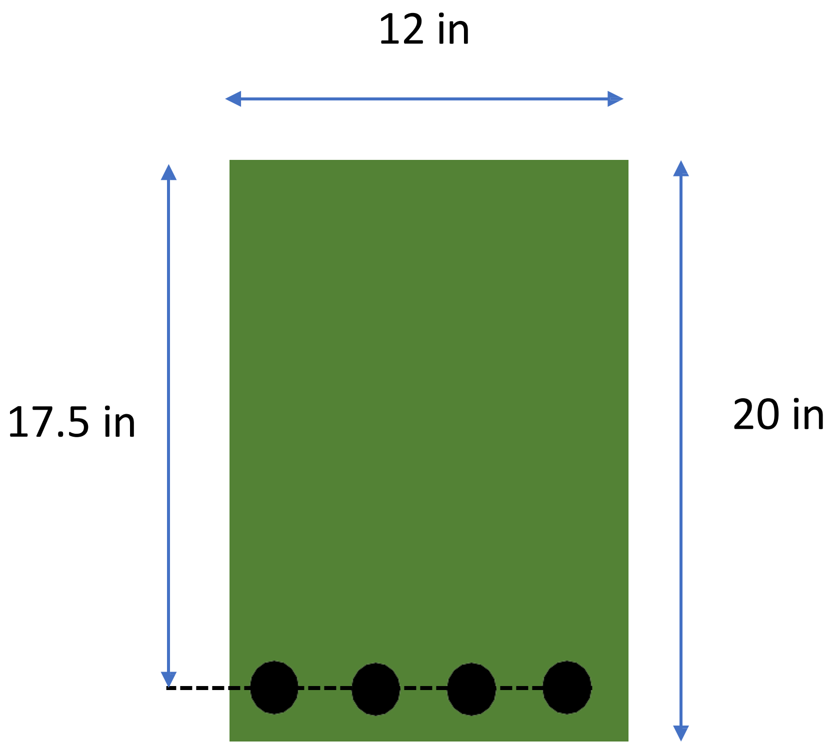
Here,

b : 12 in

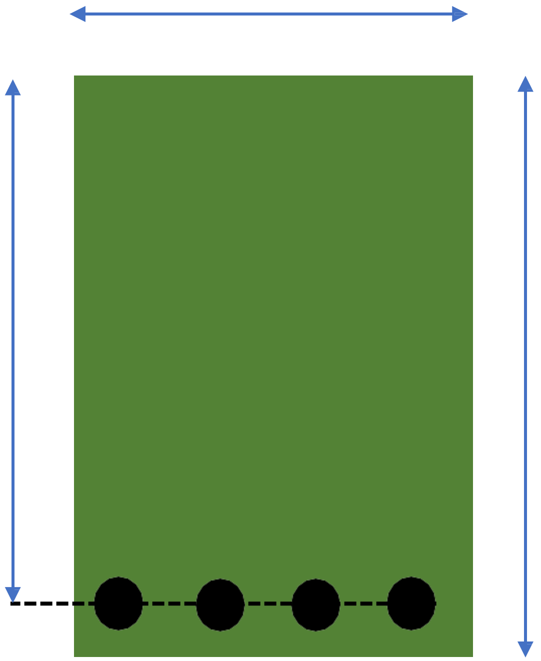
h : 20 in

d : (20-2.5) = 17.5 in

A_s : (4 x 1) = 4 in^2



A simply supported rectangular beam has a total cross section of 12 in × 20 in. It is reinforced with 4#9 bars in one row. What is the nominal flexural strength and what is the maximum moment that can be utilized in design.
Assume fc' = 4000 psi and f_y = 60000 psi.



Solution:

$$\rho = \frac{A_s}{bd} = \frac{4}{12 \times 17.5} = 0.019$$

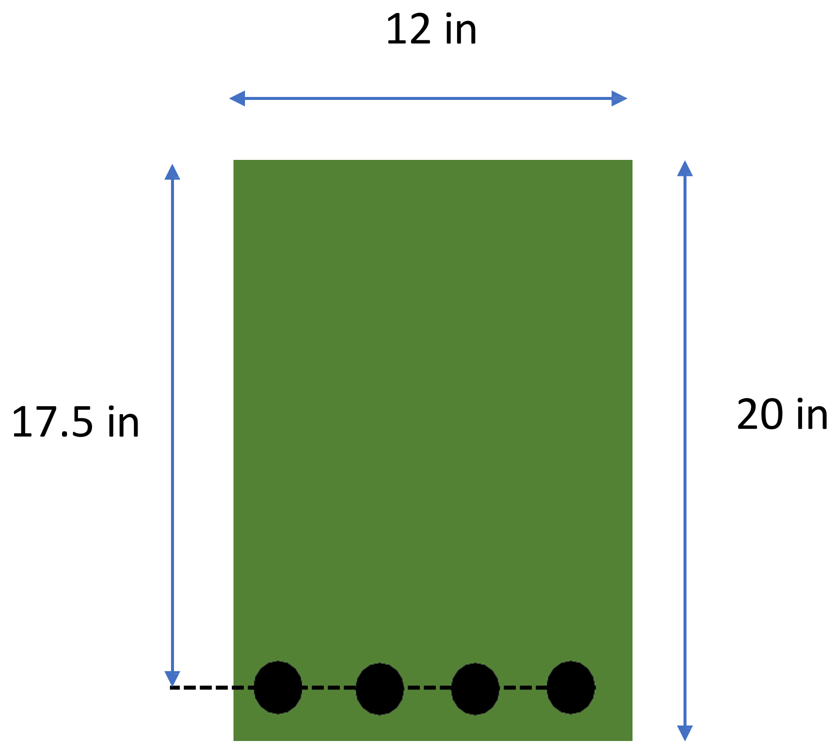
$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$$= 0.85 \times 0.85 \times \frac{4000}{60000} \frac{0.003}{0.003 + \frac{60000}{29000000}}$$

$$= 0.0289$$

Since $\boldsymbol{\rho} < \boldsymbol{\rho_b}$, the beam section is **under-reinforced**.

A simply supported rectangular beam has a total cross section of 12 in $\times$ 20 in. It is reinforced with 4#9 bars in one row. What is the nominal flexural strength and what is the maximum moment that can be utilized in design.
Assume fc' = 4000 psi and f_y = 60000 psi.



Solution:

$$a = \frac{A_s f_y}{0.85 \times fc' \times b} = \frac{4 \times 60000}{0.85 \times 4000 \times 12} = 5.88\ in$$

$Nominal\ moment, M_n$

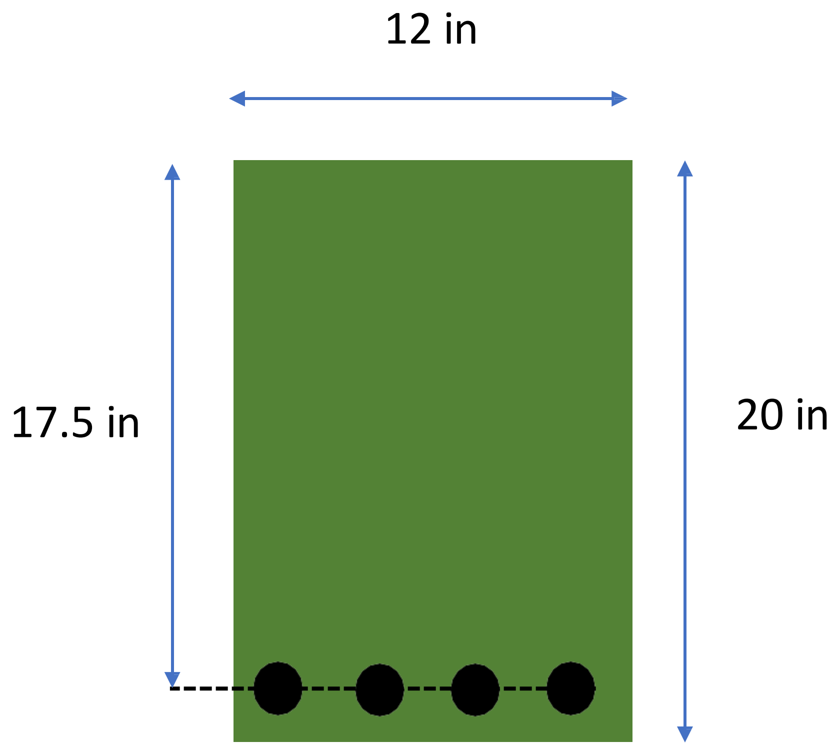
$$= A_s f_y (d - \frac{a}{2})$$

$$= 4 \times 60000 \times (17.5 - \frac{5.88}{2})$$

$$= 3494400\ lb - in$$

$$= \mathbf{291\ k - ft}\ \text{(Ans)}$$

A simply supported rectangular beam has a total cross section of $12\text{ in} \times 20\ in$. It is reinforced with 4#9 bars in one row. What is the nominal flexural strength and what is the maximum moment that can be utilized in design.
Assume fc' = 4000 psi and f_y = 60000 psi.



Solution:

$$c = \frac{a}{\beta_1} = \frac{5.88}{0.85} = 6.92\ in$$

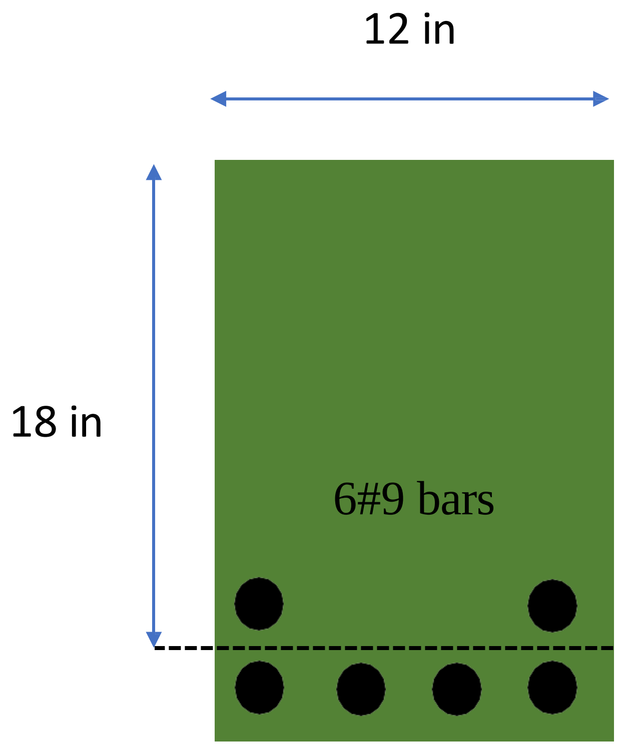
$$Net\ tensile\ strain, \varepsilon_t = \varepsilon_u \frac{d-c}{c}$$

$$= 0.003 \times \frac{17.5 - 6.92}{6.92} = 0.0046$$

$$\Phi = 0.483 + 83.3 \times \varepsilon_t = 0.87$$

$$M_u = \Phi M_n = 0.87 \times 291 = \mathbf{253\ k - ft}\ \text{(Ans)}$$

Analysis 02: A beam section is shown in figure below. Calculate nominal and ultimate moment. Assume fc' = 3000 psi and f_y = 60000 psi.



Solution:

Here,

b : 12 in

d : 18 in

A_s : (6 x 1) = 6 in^2

$$\rho = \frac{A_s}{bd} = \frac{6}{12 \times 18} = 0.0278$$

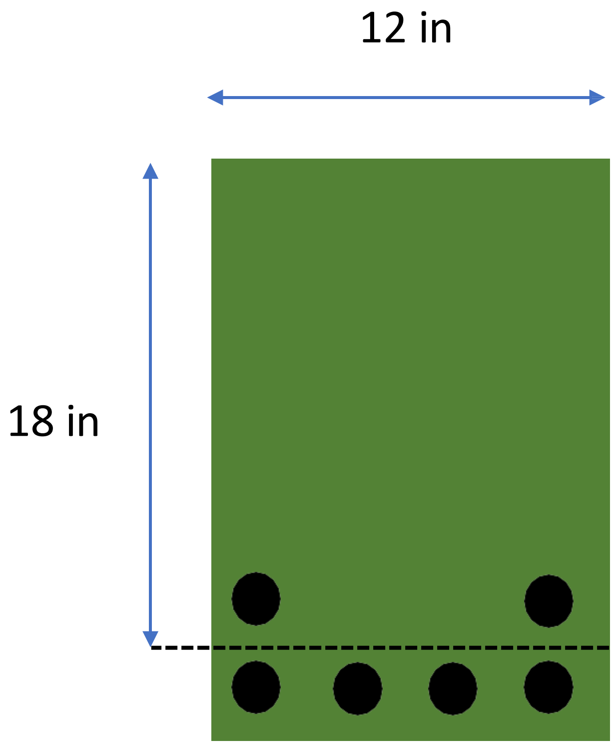
A beam section is shown in figure below. Calculate nominal and ultimate moment. Assume fc’ = 3000 psi and f_y = 60000 psi.

Solution:

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$$= 0.85 \times 0.85 \times \frac{3000}{60000} \frac{0.003}{0.003 + \frac{60000}{29000000}}$$

$$= 0.0217$$



Since $\boldsymbol{\rho} > \boldsymbol{\rho_b}$, the beam section is **over-reinforced**.

A beam section is shown in figure below. Calculate nominal and ultimate moment. Assume fc' = 3000 psi and f_y = 60000 psi.

Solution:

At equilibrium,

$C = T$

$\alpha f'_c bc = A_s f_s$

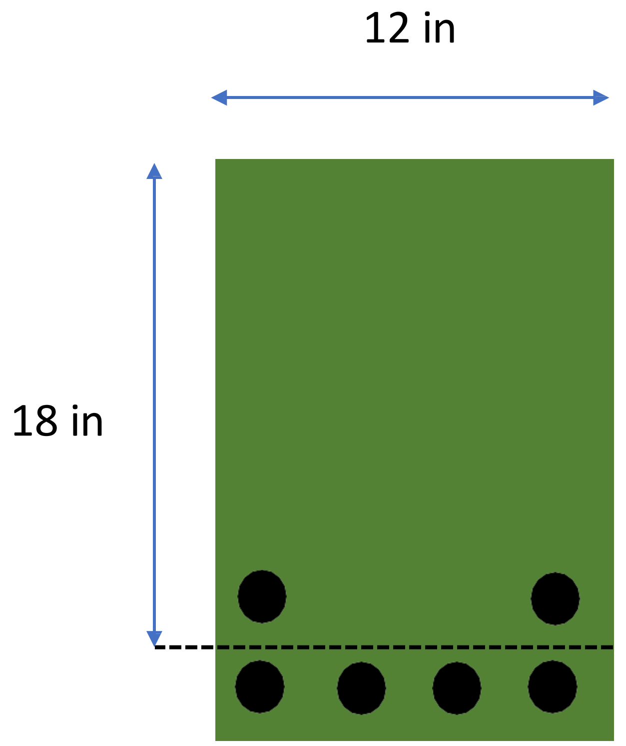
$0.85 f'_c \beta_1 bc = 6 \times E_u \times \varepsilon_u \dfrac{d-c}{c}$

$24480 \times c = 6 \times 29000000 \times 0.003 \times \frac{18-c}{c}$

$c = 11.46\ in$

$\dfrac{a}{\beta_1} = 11.46$

$a = 11.46 \times 0.85 = 9.74\ in$



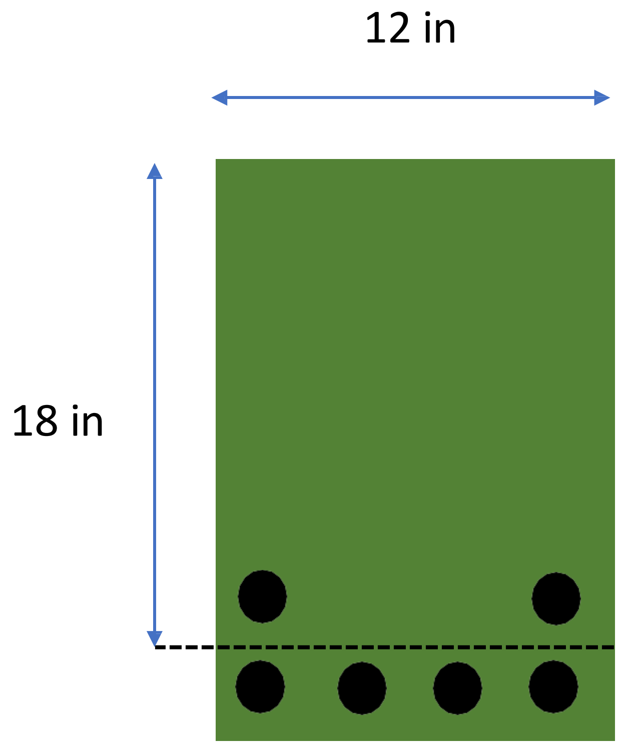
A beam section is shown in figure below. Calculate nominal and ultimate moment. Assume fc' = 3000 psi and f_y = 60000 psi.

Solution:

$$f_s = 6 \times E_u \times \varepsilon_u \frac{d - c}{c}$$

$$= 6 \times 29000000 \times 0.003 \times \frac{18-11.46}{11.46}$$

$$= 49649.21\, psi$$



A beam section is shown in figure below. Calculate nominal and ultimate moment. Assume fc' = 3000 psi and f_y = 60000 psi.

Solution:

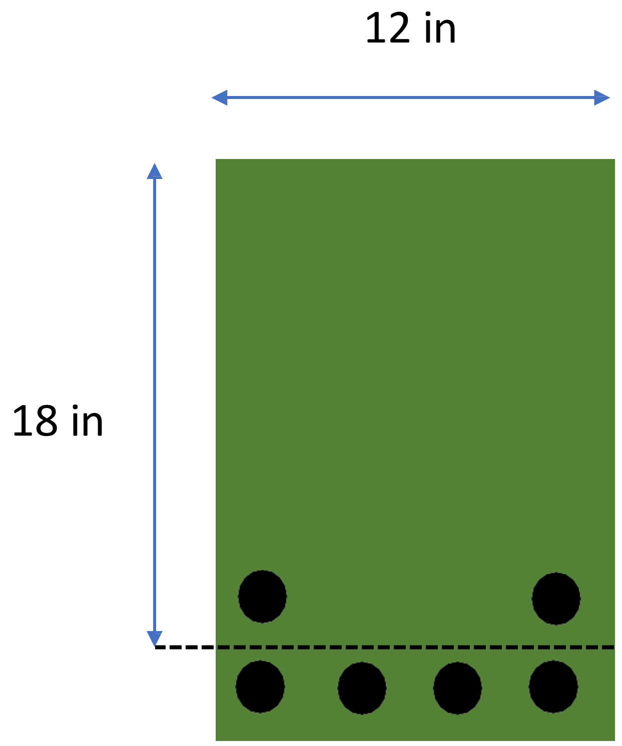
$$Nominal\ moment, M_n$$

$$= A_s f_s(\mathrm{d} - \frac{a}{2})$$

$$= 6 \times 49649.21 \times (18 - \frac{9.74}{2})$$

$$= 3911364.76\ lb - in$$

$$= \mathbf{391\ k - ft}\ (\text{Ans})$$



A beam section is shown in figure below. Calculate nominal and ultimate moment. Assume fc’ = 3000 psi and f_y = 60000 psi.

Solution:

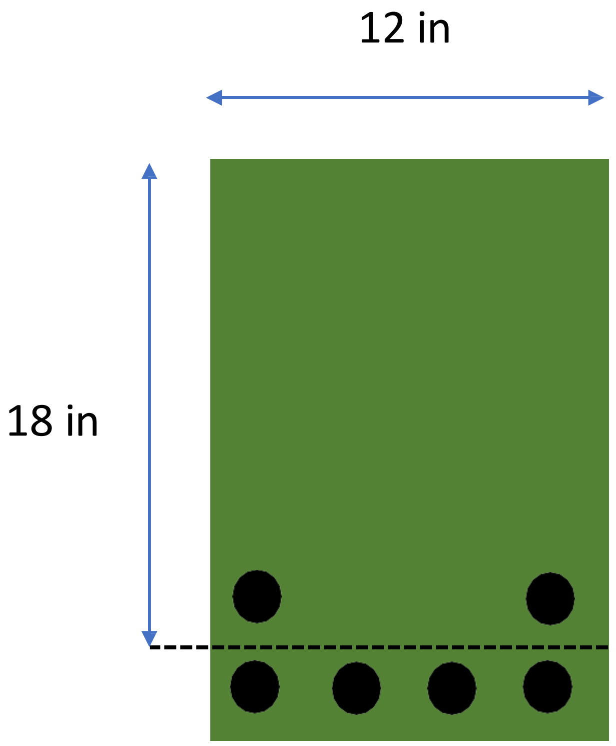
$$c = \frac{a}{\beta_1} = \frac{9.74}{0.85} = 11.46\ in$$

$$Net\ tensile\ strain, \varepsilon_t = \varepsilon_u \frac{d-c}{c}$$

$$= 0.003 \times \frac{18-11.46}{11.46} = 0.0017$$

Since $\varepsilon_t < 0.002$, $\Phi = 0.65$

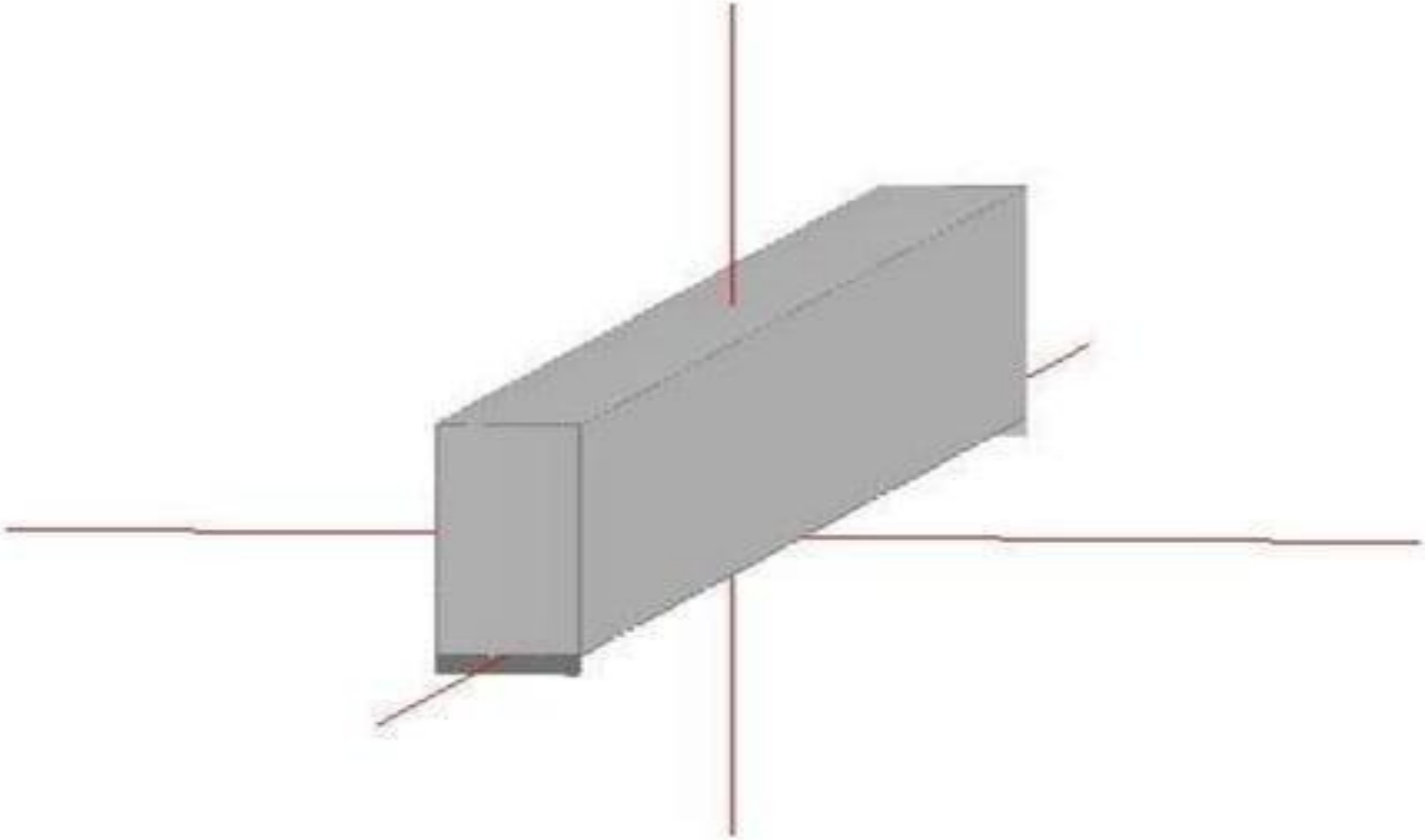
$$M_u = \Phi M_n = 0.65 \times 391 = \mathbf{254\ k-ft}\ \text{(Ans)}$$



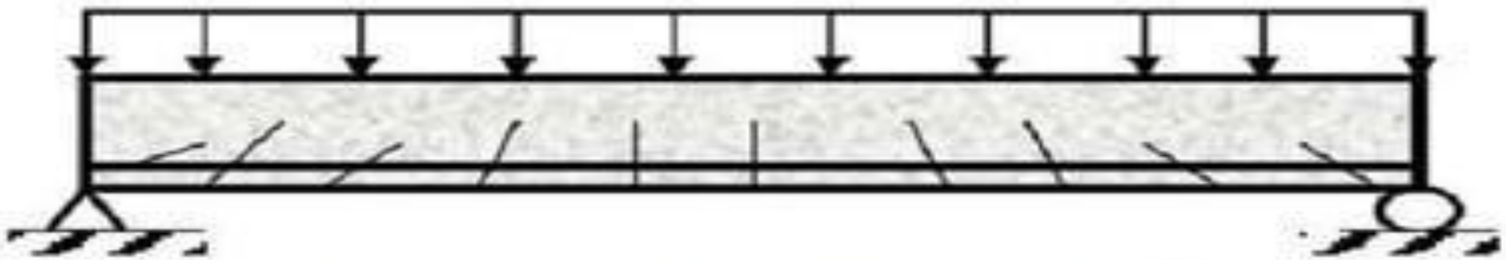
Design of Singly Reinforced Beam (WSD)

WEEK-05

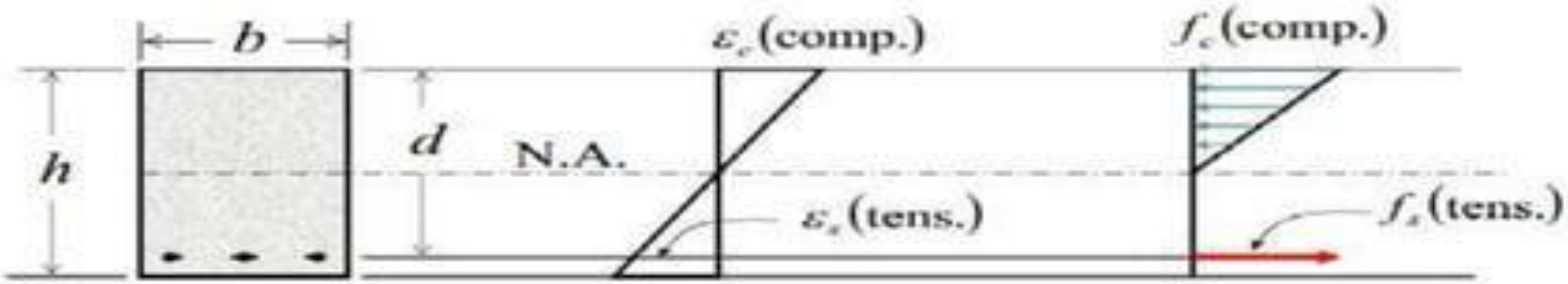
Section of Rectangular Beam



Stress distribution in rectangular beam under working load



Reinforced Concrete Beam



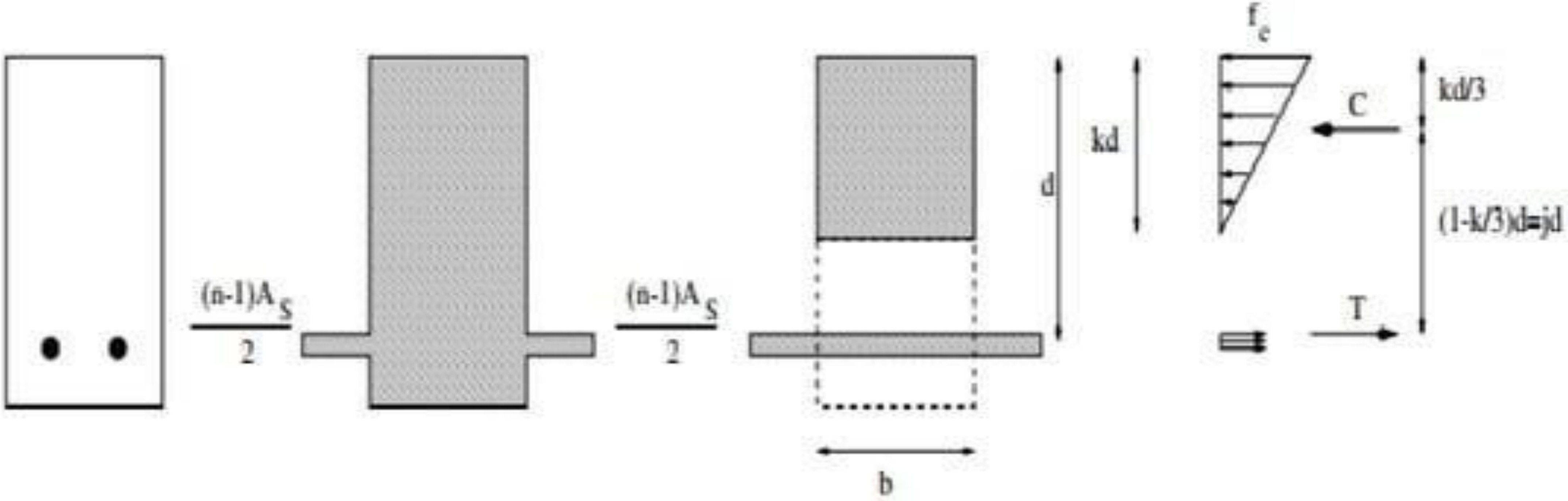
Singly Rectangular Beam

- A Beam is any structural member which resists load mainly by bending. Therefore it is also called flexural member. Beam may be singly reinforced or doubly reinforced. When steel is provided only in tensile zone (i.e. below neutral axis) is called **singly reinforced beam.**

Assumptions for DesignofMembers by Working Stress Method

- Plane sections before bending remain plane after bending.
- Normally, concrete is not considered for taking the tensile stresses except otherwise specifically permitted. Therefore, all tensile stresses are taken up by reinforcement only.
- The stress-strain relationship of steel and concrete is a straight line under working loads.

Singly Rectangular Beam stress distribution



Where,

- Jd =momnet arm=(1-k/3)
- Compressive force ,

  C =(bkd/2)fc
- Tensile force ,

  T = Asfs

- neutral axis is determined by equating the moment of the tension area to the moment of the compression area

$$b(kd)\left(\frac{kd}{2}\right) = nA_s(d-kd)$$

$$M = Tjd = A_s f_s jd \Rightarrow f_s = \frac{M}{A_s jd}$$

$$M = Cjd = \frac{bkd}{2} f_c jd = \frac{bd^2}{2} kj f_c \Rightarrow \boxed{f_c = \frac{M}{\frac{1}{2}bd^2 kj}}$$

Moment calculation:

- Start by determinir ρ;

- If $\rho < \rho_b$ steel reaches max. allowable value before concrete, and

$$\boxed{M = A_s f_s j d}$$

- If $\rho > \rho_b$ concrete reaches max. allowable value before steel and

$$M = f_c \frac{bkd}{2} jd$$

or

$$\boxed{M = \frac{1}{2} f_c j k b d^2 = R b d^2}$$

Reinforcement calculation:

Design We define

$$R \stackrel{\text{def}}{=} \frac{1}{2} f_c k j$$

where $k = \frac{n}{n+r}$, solve for bd^2 from

$$bd^2 = \frac{M}{R}$$

assume b and solve for d. Finally we can determine A_s from

$$A_s = \rho_b bd$$

Design equation for singly reinforcement beam

Review	Design
$b, d, A_s \checkmark$ $M?$	$M \checkmark$ $b, d, A_s?$
$\rho = \frac{A_s}{bd}$ $k = \sqrt{2\rho n + (\rho n)^2} - \rho n$ $r = \frac{f_s}{f_c}$ $\rho_b = \frac{n}{2r(n+r)}$ $\rho < \rho_b \quad M = A_s f_s j d$ $\rho > \rho_b \quad M = \frac{1}{2} f_c b k d^2 j$	$k = \frac{n}{n+r}$ $j = 1 - \frac{k}{3}$ $r = \frac{f_s}{f_c}$ $R = \frac{1}{2} f_c k j$ $\rho_b = \frac{n}{2r(n+r)}$ $bd^2 = \frac{M}{R}$ $A_s = \rho_b bd$ or $A_s = \frac{M}{f_s j d}$

Design of Singly Reinforced Beam (USD)

WEEK-06

EVALUATION OF DESIGN PARAMETERS

- Total compressive force - $C = 0.85fc' ba$ (Refer stress diagram)

- Total Tensile force-

$$T = As\ fy$$
$$C = T$$
$$0.85fc'\ ba = As\ fy$$
$$a = As\ fy\ /\ (0.85fc'\ b)$$
$$= \rho d\ fy\ /\ (0.85\ fc')\ [\therefore\ \rho = As\ /\ bd]$$

- Moment of Resistance/Nominal Moment-

$$Mn = 0.85fc'\ ba\ (d - a/2) \quad \text{or,}$$
$$Mn = As\ fy\ (d - a/2)$$
$$= \rho\ bd\ fy\ [\ d - (\rho dfyb\ /\ 1.7fc')\ ]$$
$$= \omega\ fc'\ [\ 1 - 0.59\ \omega]\ bd^2$$
$$\therefore\ \omega = \rho\ fy\ /\ fc'$$
$$Mn = Kn\ bd^2 \quad \therefore Kn = \omega\ fc'\ [\ 1 - 0.59\ \omega]$$

Ultimate Moment- $Mu = \phi\ Mn$

$$= \phi\ Kn\ bd^2$$

Balaced Reinforcement Ratio (ρ_b)

From strain diagram, similar triangles

$c_b / d = 0.003 / (0.003 + fy / Es)$; $Es = 29\times10^6$ psi

$c_b / d = 87{,}000 / (87{,}000 + fy)$

Relationship b / n the depth `a' of the equivalent rectangular stress block & depth `c' of the N.A. is

$a = \beta_1 c$

$\beta_1 = 0.85$	; $fc' \leq 4000$ psi
$\beta_1 = 0.85 - 0.05(fc' - 4000) / 1000$	; $4000 < fc' \leq 8000$
$\beta_1 = 0.65$	; $fc' > 8000$ psi

$\rho_b = A_{sb} / bd$

$= 0.85 fc' \, a_b / (fy. \, d)$

$= \beta_1 \, (0.85 \, fc' / fy) \, [\, 87{,}000 / (87{,}000 + fy)]$

- For beams the ACI code limits the max. amount of steel to 75% of that required for balanced section.

$$\rho \leq 0.75\, \rho_b$$

- Min. reinforcement is greater of the following:

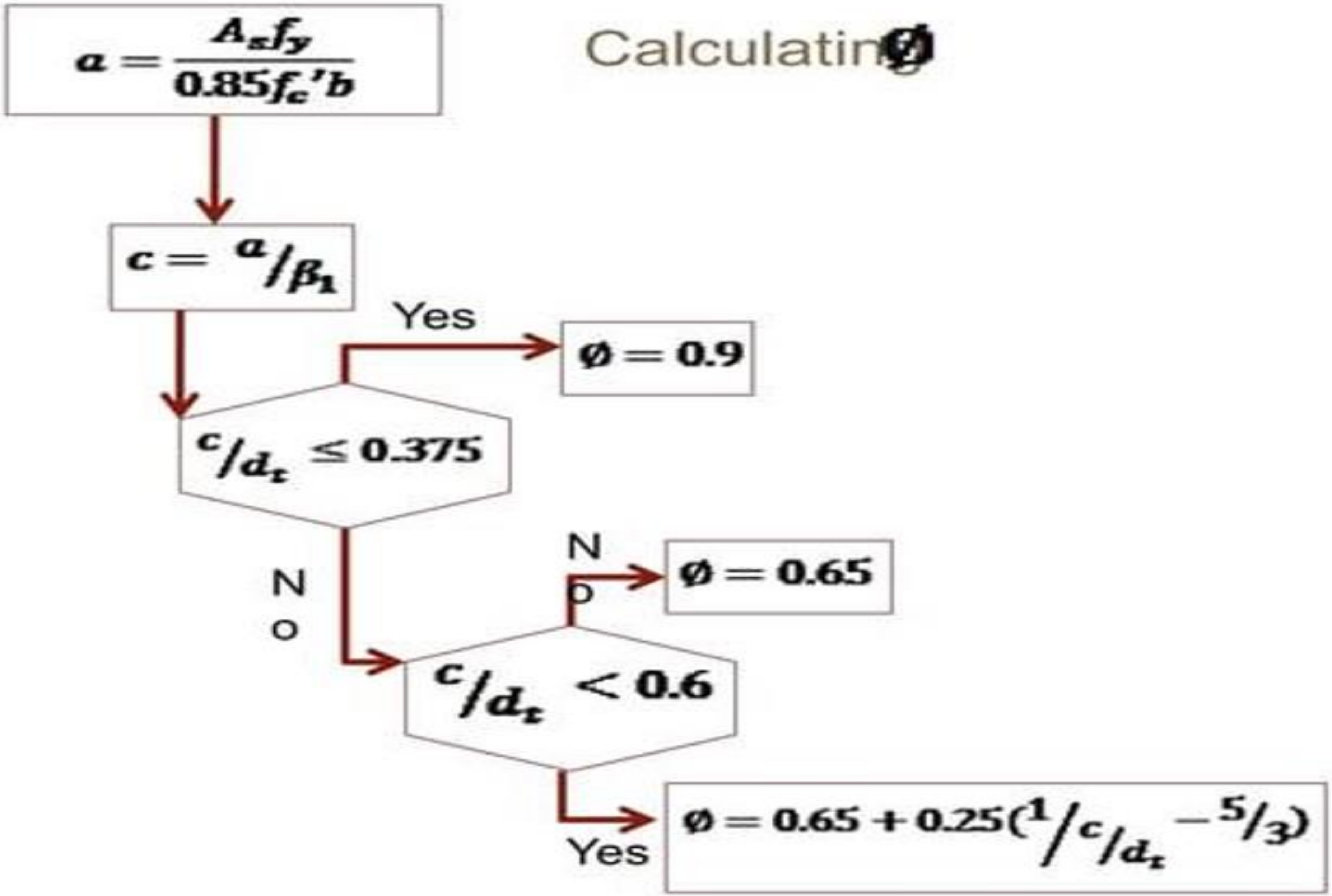
$As_{min} = 3\sqrt{fc'} \times b_w d / fy$ or $200\, b_w d / fy$

$\rho_{min} = 3\sqrt{fc'} / fy$ or $200 / fy$

- For statically determinate member, when the flange is in tension, the b_w is replaced with $2b_w$ or bf whichever is smaller

- The above min steel requirement need not be applied, if at every section, **Ast** provided is at least 1/3 greater than the analysis

Calculating Ø



Calculating β_1

$$f'_c \leq 4000\,psi$$

$$\beta_1 = 0.85 - 0.05\,\frac{f'_c - 4000}{1000} \geq 0.65$$

$$\beta_1 = 0.85$$

SINGLY REINFORCED BEAM

- Beam is reinforced near the tensile face
- Reinforcement resists the tension.
- Concrete resists the compression.

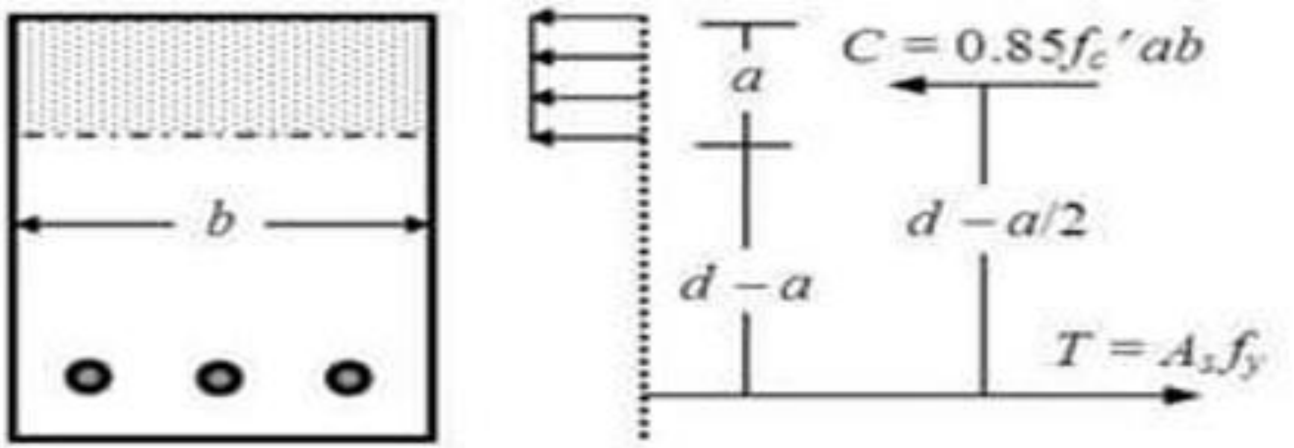


Fig. 6.2: Forces from Rectangular Stress Block

DESIGN PROCEDURE FOR SINGLY REINFORCED BEAM

1. Determine the service loads

2. Assume `h` as per the support conditions [As per ACI code in table 9.5(a)]

3. Calculate d= h – Effective cover

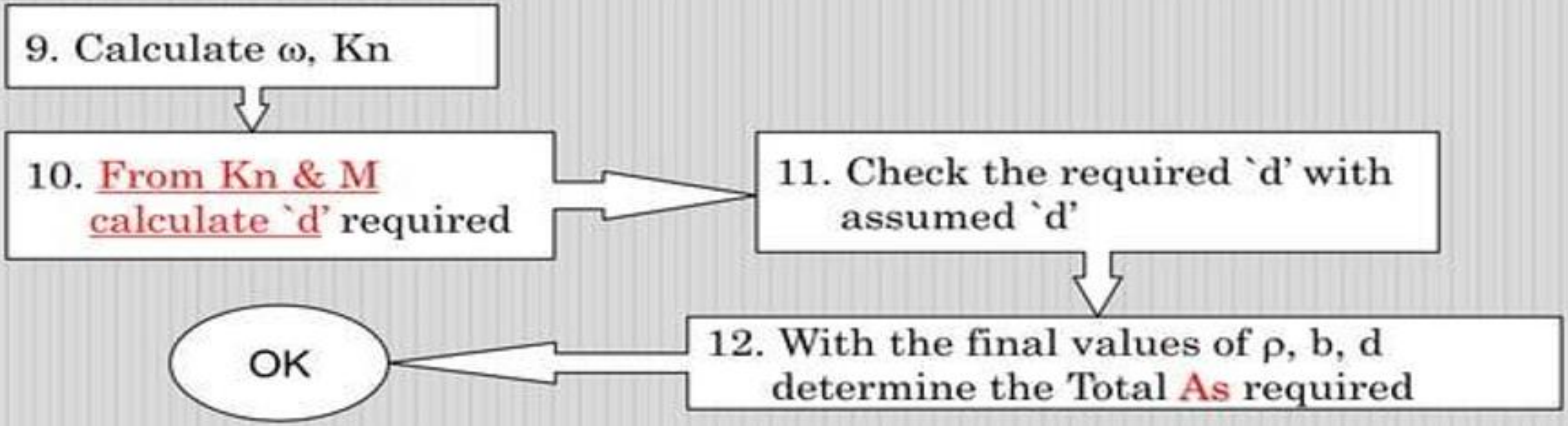
4. Assume the value of `b` by the rule of thumb

5. Estimate self weight

6. Primary elastic analysis and derive B.M (M), Shear force (V) values

7. Compute ρ_{min} and ρ_b

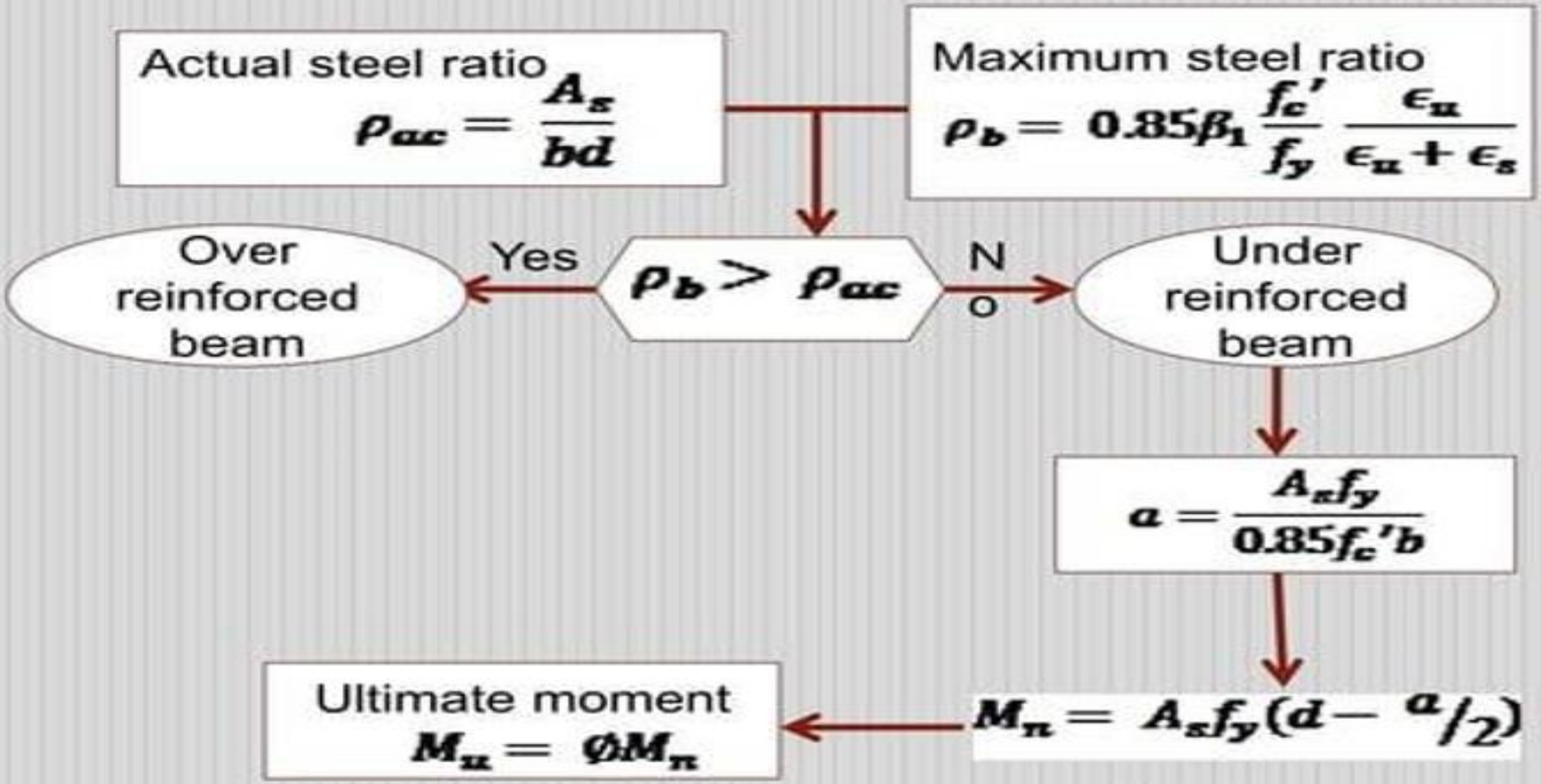
8. Choose ρ between ρ_{min} and ρ_b



Note Below:

- **Design the steel reinforcement arrangement with appropriate cover and spacing stipulated in code. Bar size and corresponding no. of bars based on the bar size #n.**
- **Check crack width as per codal provisions.**

DESIGN PROCEDURE FOR SINGLY REINFORCED BEAM BY FLOWCHART



Design of Doubly Reinforced Beam (WSD)

WEEK-07

Doubly Reinforcement beam

- If a beam cross section is limited because of architectural or other considerations, it may happen that the concrete can not develop the compression force required to resist the given bending moment, in this case ,reinforcement is added in the compression zone, resulting in so called **Doubly reinforced beam** .i.e., one with compression as well as tension reinforcement.

Why Doubly reinforcement used

- If concrete can not develop the required compressive force to resist the maximum bending moment, then additional reinforcement is provide in the compression zone.
- Reinforcement is provided in both compression and tension zone.

Development of moment

- In stress distibution,the design moment is more than the balanced moment of resistance of the section,

$$M = M_1 + M_2$$

The additional moment M_2 is resisted by providing compression reinforcement As'

and additional tensile reinforcement As_2

Moment calculation

- Moment is obtained by,

$$M_1=(fc/2)kjbd^2$$
$$= As_1fsjd$$

$$M_2=As_2fs(d-d')$$
$$=As'fs'(d-d')$$

Reinforcement in Beam

- The total tensile reinforcement Ast has two components As_1+As_2 for M_1 and M_2
- The equation of Ast,

$$Ast=\hat{\rho}\text{ş}1+As2$$

Where, $As_1=$ (bd/100)

And $As_2=$ M2/[Ó (d-d')]

- compressive reinforcement,

$$As'=M_2/[fs'(d-d')]$$

Clear cover for design

- **Not less than 1.5 in. when there is no exposure to weather or contact with the ground**
- **For exposure to aggressive weather 2 in.**

CLEAR DISTENCE OF REINFORCEMENT

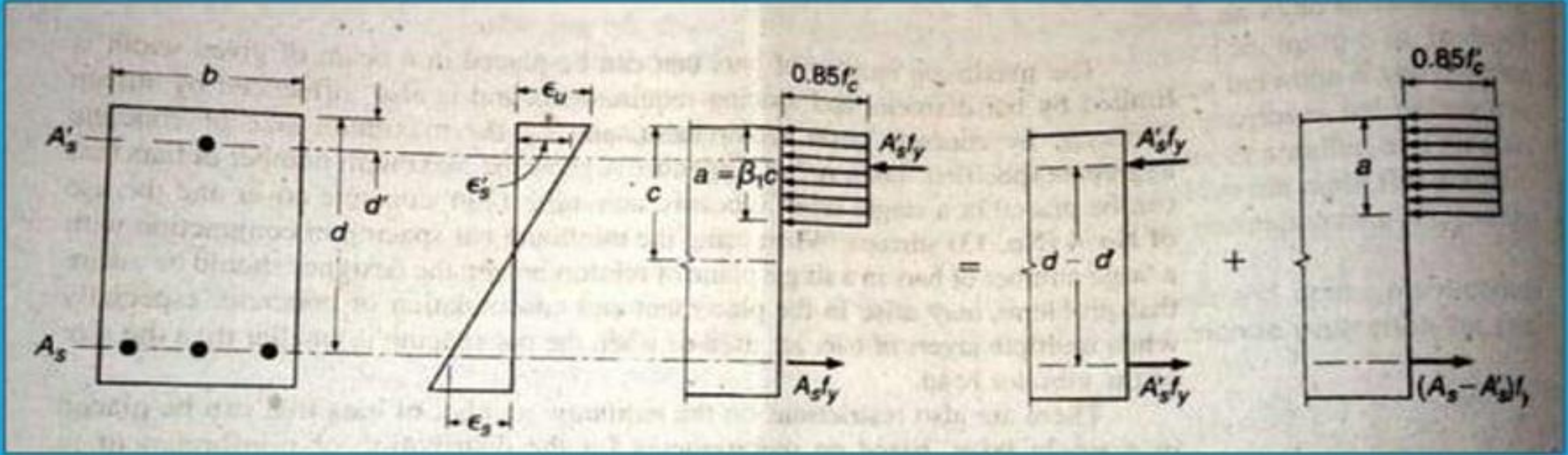
- **Clear distance between parallel bars in a layer must not be less than the bar diameter or 1 in.**

Design of Doubly Reinforced Beam (USD)

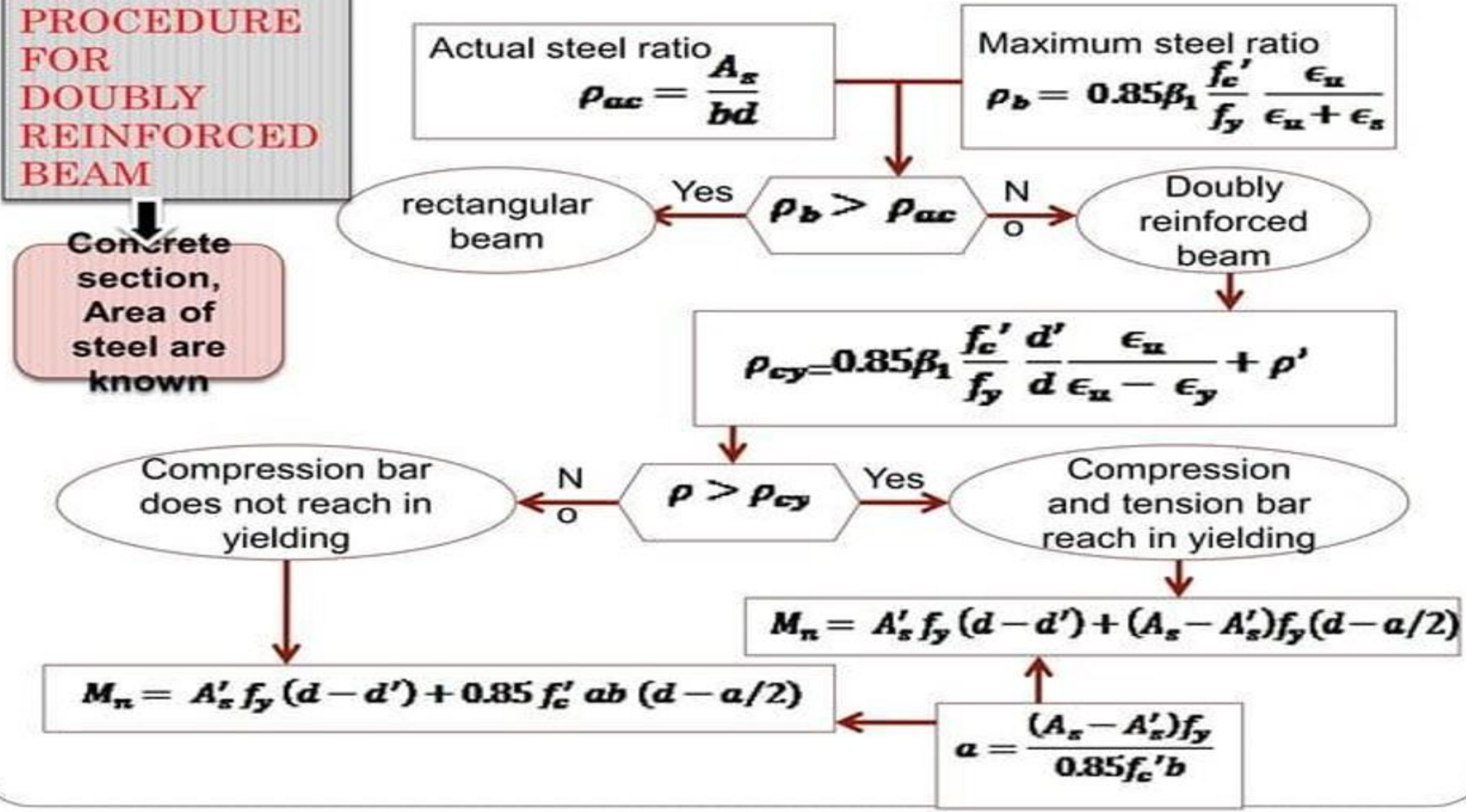
WEEK-08

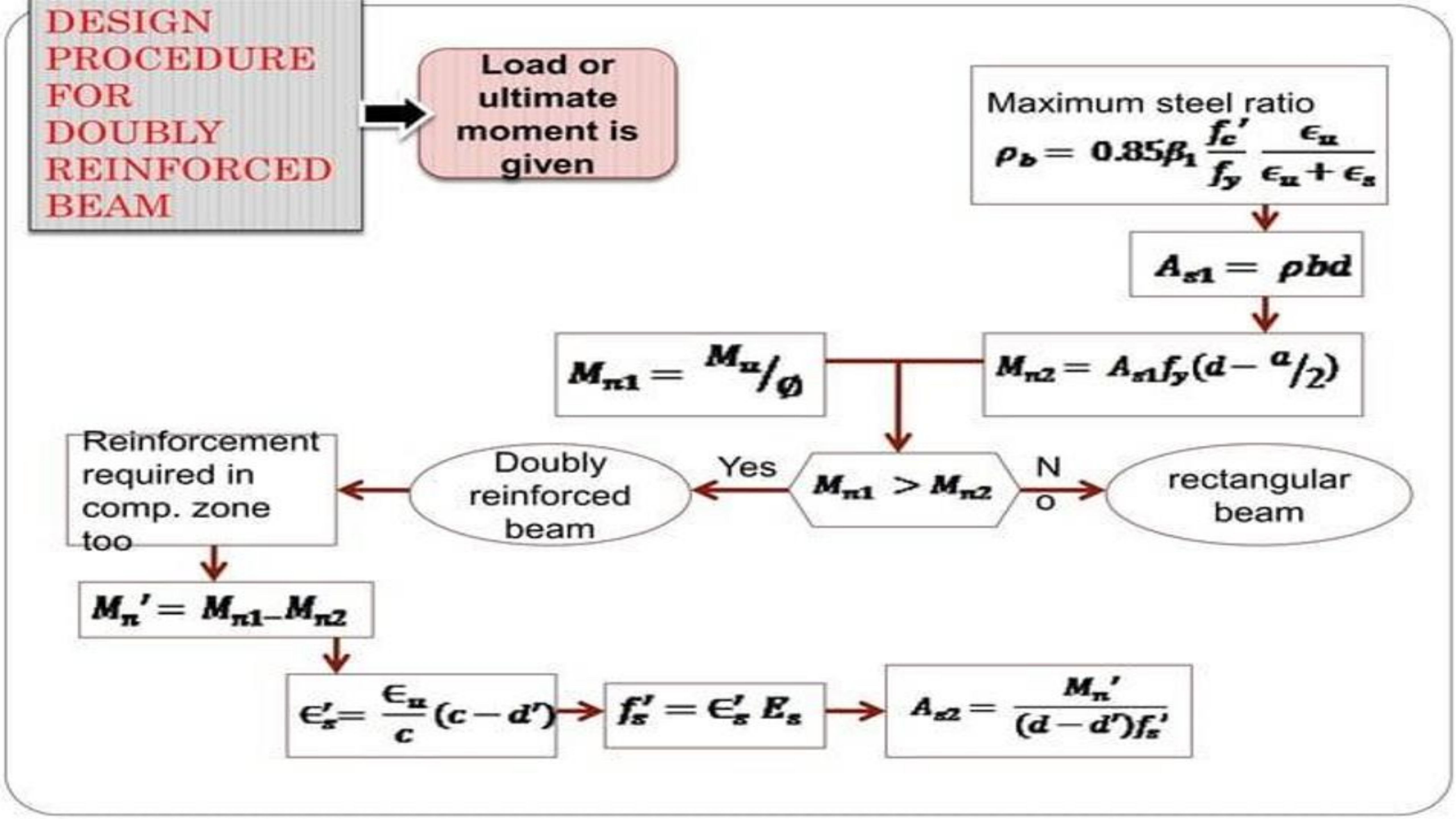
DOUBLY REINFORCED BEAM

- Beam is fixed for Architectural purposes.
- Reinforcement are provided both in tension and compression zone.
- Concrete has limitation to resist the total compression so extra reinforcement is required.



DESIGN PROCEDURE FOR DOUBLY REINFORCED BEAM




DESIGN PROCEDURE FOR DOUBLY REINFORCED BEAM
Load or ultimate moment is given
Maximum steel ratio
$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_s}$
$A_{s1} = \rho b d$
$M_{n1} = M_u/\phi$
$M_{n2} = A_{s1} f_y (d - a/2)$
$M_{n1} > M_{n2}$
Yes
No
Doubly reinforced beam
rectangular beam
Reinforcement required in comp. zone too
$M_n' = M_{n1} - M_{n2}$
$\epsilon_s' = \frac{\epsilon_u}{c}(c - d')$
$f_s' = \epsilon_s' E_s$
$A_{s2} = \frac{M_n'}{(d - d') f_s'}$

Analysis of Doubly Reinforced Beam (USD)

WEEK-09

Analysis of Doubly Reinforced Beams

- Assume tension steel has yielded
- Assume compression steel has yielded
- Compute stress block depth *a* and neutral axis depth *c*
- Compute compression and tension steel strains
- If compression and tension steel have yielded, OK continue, perform strain checks and compute nominal moment
- If compression steel has not yielded, express unknown stress block depth and compression steel stress in terms of neutral axis depth *c* and solve resulting quadratic equation. Perform strain checks and compute nominal moment.
- If tension steel has not yielded, use adapted method

Analysis of Doubly Reinforced Beams
(Compression steel has yielded)

Assume compression and tension steel have yielded : $\Rightarrow f_s = f_s^{'} = f_y$

Compute stress block depth from eq.(a) :

$$a = \frac{A_s f_y - A_s^{'}\left(f_s^{'} - 0.85 f_c^{'}\right)}{0.85 f_c^{'} b} = \frac{A_s f_y - A_s^{'}\left(f_y - 0.85 f_c^{'}\right)}{0.85 f_c^{'} b} \Rightarrow c = \frac{a}{\beta_1}$$

Steel strains: $\varepsilon_s^{'} = 0.003 \dfrac{c - d^{'}}{c}$, $\varepsilon_s = 0.003 \dfrac{d - c}{c}$

If compression and tension steel have yielded, as assumed, then :

compute nominal moment with eq.(b) :

$$M_n = 0.85 f_c^{'} ab\left(d - \frac{a}{2}\right) + A_s^{'}\left(f_y - 0.85 f_c^{'}\right)\left(d - d^{'}\right)$$

Analysis of Doubly Reinforced Beams
(Compression steel has not yielded) (1)

If compression steel has not yielded: $\varepsilon_s' < \varepsilon_y \Rightarrow f_s' = E_s \varepsilon_s'$

$$f_s' = E_s \varepsilon_s' = 200000 \times 0.003 \frac{c-d'}{c} = 600 \frac{c-d'}{c} < f_y$$

Back to force equilibrium equation:

$$T = C_c + C_s \Rightarrow A_s f_y = 0.85 f_c' ab + A_s' \left(f_s' - 0.85 f_c' \right)$$

Substituting the stress block depth ($a = \beta_1 c$) and the compression steel stress in terms of neutral axis depth gives:

$$A_s f_y = 0.85 f_c' b \beta_1 c + A_s' \left(600 \frac{c-d'}{c} - 0.85 f_c' \right)$$

$$\Rightarrow 0.85 f_c' b \beta_1 c + 600 A_s' \frac{c-d'}{c} - 0.85 f_c' A_s' - A_s f_y = 0$$

Analysis of Doubly Reinforced Beams (Compression steel has not yielded) (2)

Multiplying all terms by *c* and assembling leads to a quadratic equation with respect to the neutral axis depth *c* :

$$c^2 - \left(P - P' + R'\right) \times c - P'd' = 0$$

with $P = \dfrac{A_s f_y}{0.85 f_c' b \beta_1}$ $\quad P' = \dfrac{600 A_s'}{0.85 f_c' b \beta_1}$ $\quad R' = \dfrac{A_s'}{b \beta_1}$

The positive solution is: $c = \dfrac{\left(P - P' + R'\right)}{2}\left(\sqrt{1 + \dfrac{4P'd'}{\left(P - P' + R'\right)^2}} + 1\right)$

Compute steel strains and check tension yield and no compression yield

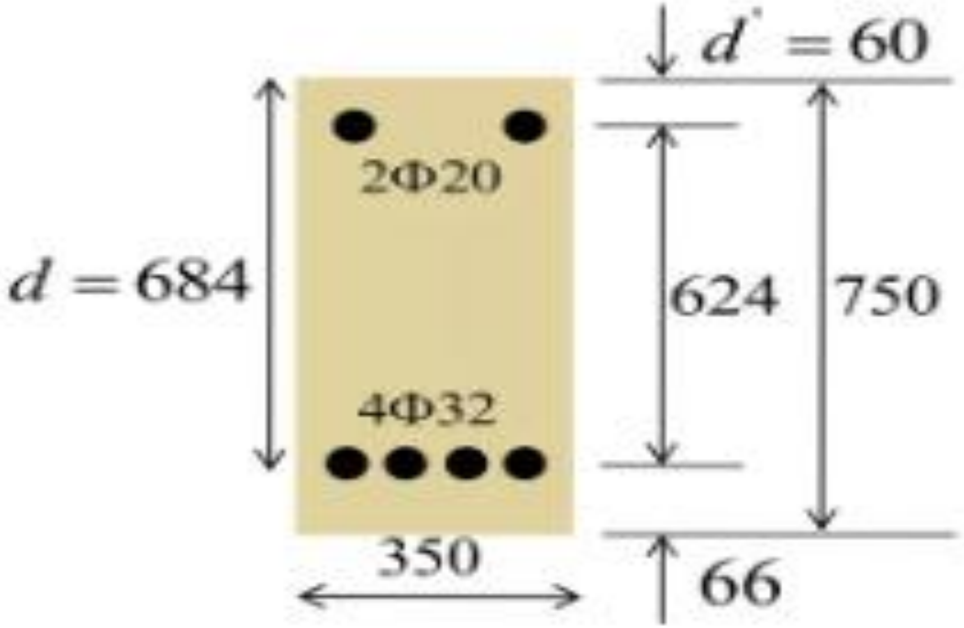
If OK: $M_n = 0.85 f_c' ab\left(d - \dfrac{a}{2}\right) + A_s'\left(f_s' - 0.85 f_c'\right)\left(d - d'\right)$ **eq.(b)**

Analysis Problem-1

Determine the design moment capacity of the beam shown

$f_y = 420\,\text{MPa}$ and $f_c' = 20\,\text{MPa}$

All dimensions in mm



$$A_s = 4\frac{\pi \times 32^2}{4} = 3217.0\,mm^2$$

$$A_s' = 2\frac{\pi \times 20^2}{4} = 628.3\,mm^2$$

$$d = h - \left(\text{cover} + d_s + \frac{d_b}{2}\right) = 750 - \left(40 + 10 + \frac{32}{2}\right) = 750 - 66 = 684\,mm$$

$$d' = \text{cover} + d_s + \frac{d_b'}{2} = 40 + 10 + \frac{20}{2} = 60\,mm$$

Solution 1

$$\text{Assume compression steel has yielded:} \quad \varepsilon_s^{'} \geq \varepsilon_y \Rightarrow f_s^{'} = f_y$$

$$\text{Compute stress block depth:} \quad a = \frac{A_s f_y - A_s^{'}\left(f_y - 0.85 f_c^{'}\right)}{0.85 f_c^{'} b}$$

$$a = \frac{3217.0 \times 420 - 628.3(420 - 0.85 \times 20)}{0.85 \times 20 \times 350} = 184.527\, mm$$

$$\text{N.A. depth:} \quad c = \frac{a}{\beta_1} = \frac{184.527}{0.85} = 217.09\, mm$$

$$\text{Compression steel strain:} \quad \varepsilon_s^{'} = 0.003 \frac{c - d^{'}}{c} = 0.003 \frac{217.09 - 60}{217.09}$$

$$\varepsilon_s^{'} = 0.00217 \qquad \varepsilon_s^{'} > \varepsilon_y = 0.0021 \Rightarrow \text{OK assumed yielding}$$

$$\text{Tension strain:} \quad \varepsilon_t = 0.003 \frac{d_t - c}{c} = 0.003 \frac{684 - 217.09}{217.09}$$

$$\varepsilon_t = 0.00645 \quad \varepsilon_t \geq 0.005 \Rightarrow \text{OK tension-control}$$

Solution 1 – Cont.

Nominal moment using eq.(b):

$$M_n = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s'\left(f_s' - 0.85 f_c'\right)\left(d - d'\right)$$

$$M_n = 0.85 \times 20 \times 184.527 \times 350\left(684 - \frac{184.527}{2}\right)$$

$$+ 628.3(420 - 0.85 \times 20)(684 - 60)$$

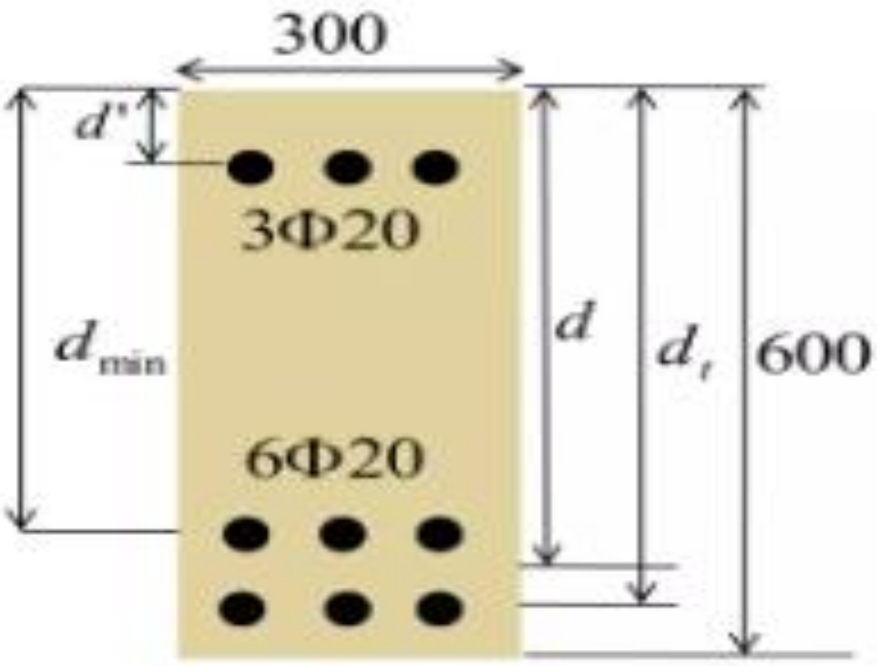
$$M_n = 807.7 \times 10^6\ N.mm = 807.7\ kN.m$$

Design moment $\phi M_n = 0.90 \times 807.7 = 726.9\ kN.m$

Analysis Problem-2

Analysis of a rectangular section 300 x 600 mm with six 20-mm bars in two layers as tension steel and three 20-mm bars as compression reinforcement. Net spacing between steel layers is 30 mm. Stirrups have 10-mm diameter

$f_c' = 25\,MPa \qquad f_y = 420\,MPa$



$$d' = 40 + 10 + 10 = 60\,mm$$

$$d_t = d_1 = h - 60 = 540\,mm$$

$$d_2 = d_1 - S_t - 20 = 540 - 50 = 490\,mm$$

$$d_{min} = d_2 = 490\,mm$$

$$d = \frac{d_1 + d_2}{2} = 515\,mm$$

$$A_s = 1884.96\,mm^2$$

$$A_s' = 942.48\,mm^2$$

Solution 2

Assume compression steel has yielded: $\varepsilon_s' \geq \varepsilon_y \Rightarrow f_s' = f_y$

Compute stress block depth: $a = \dfrac{A_s f_y - A_s'\left(f_y - 0.85 f_c'\right)}{0.85 f_c' b}$

$$a = \frac{1884.96 \times 420 - 942.48(420 - 0.85 \times 25)}{0.85 \times 25 \times 300} = 62.09\,mm$$

N.A. depth: $c = \dfrac{a}{\beta_1} = \dfrac{62.09}{0.85} = 73.05\,mm$

Compression steel strain: $\varepsilon_s' = 0.003\dfrac{c - d'}{c} = 0.003\dfrac{73.05 - 60}{73.05}$

$\varepsilon_s' = 0.000536 \qquad \varepsilon_s' < \varepsilon_y = 0.0021 \Rightarrow$ Not yielding

Solution 2 – Cont.

(Compression steel has not yielded)

Neutral axis depth is the solution of a quadratic equation :

$$c = \frac{(P - P' + R')}{2}\left(\sqrt{1 + \frac{4P'd'}{(P - P' + R')^2}} + 1\right)$$

with $P = \dfrac{A_s f_y}{0.85 f_c' b \beta_1}$ $\quad P' = \dfrac{600 A_s'}{0.85 f_c' b \beta_1}$ $\quad R' = \dfrac{A_s'}{b \beta_1}$

$$P = \frac{1884.96 \times 420}{0.85 \times 25 \times 300 \times 0.85} = 146.1007 \qquad R' = \frac{942.48}{300 \times 0.85} = 3.696$$

$$P' = \frac{600 \times 942.48}{0.85 \times 25 \times 300 \times 0.85} = 104.35765$$

$$c = \frac{(146.1007 - 104.35765 + 3.696)}{2}\left(\sqrt{1 + \frac{4 \times 104.35765 \times 60}{(146.1007 - 104.35765 + 3.696)^2}} + 1\right)$$

$$\Rightarrow c = 105.046\, mm$$

Solution 2 – Cont.

(Compression steel has not yielded old)

$$c = 105.046\,\text{mm}$$

$$\text{Compression steel strain: } \varepsilon_s' = 0.003\frac{c-d'}{c} = 0.003\frac{105.046-60}{105.046}$$

$$\varepsilon_s' = 0.00129 \qquad \varepsilon_s' < \varepsilon_y = 0.0021 \Rightarrow \text{OK not yielding}$$

$$f_s' = 600\frac{c-d'}{c} = 600\frac{105.046-60}{105.046} = 257.293\,MPa\ (< f_y = 420\,MPa)$$

$$\text{Tension strain at minimum depth: } \varepsilon_{\min} = 0.003\frac{d_{\min}-c}{c}$$

$$\Rightarrow \varepsilon_{\min} = 0.003\frac{490-105.046}{105.046} = 0.011$$

$$\varepsilon_{\min} > \varepsilon_y \quad \text{and} \quad \varepsilon_{\min} \geq 0.005 \Rightarrow \text{OK yield and OK tension-control}$$

Solution 2 – Cont.

(Compression steel has not yielded)

Nominal moment from eq.(b) : $M_n = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s'\left(f_s' - 0.85 f_c'\right)\left(d - d'\right)$

with: $f_s' = 257.293\,MPa$ and $a = \beta_1 c = 0.85 \times 105.046 = 89.289\,mm$

$$M_n = 0.85 \times 25 \times 89.289 \times 300\left(515 - \frac{89.289}{2}\right)$$

$$+942.48(257.293 - 0.85 \times 25)(515 - 60)$$

$$M_n = 368.95 \times 10^6\ N.mm = 368.95\,kN.m$$

Design moment: $\phi M_n = 0.90 \times 368.95 = 332.06\,kN.m$

Discussion – Effect of compression steel

Nominal moment: $M_n = 368.95\,kN.m$

Design moment: $\phi M_n = 0.90 \times 368.95 = 332.06\,kN.m$

Same beam without compression steel:

Nominal moment: $M_n = 358.559\,kN.m$

Design moment: $\phi M_n = 0.90 \times 358.559 = 322.70\,kN.m$

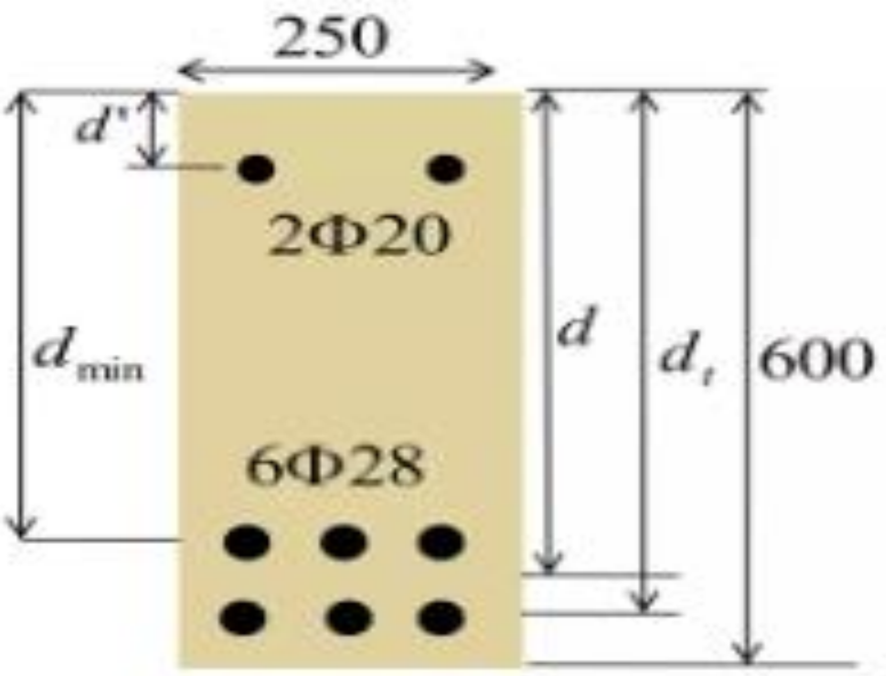
Observations about effect of compression steel :

- Three compression steel bars (50% of tension steel) have added 9.36 kN.m (2.9%) only to the beam flexural capacity, because of the reduced stress (61 % of yield value).
- Little advantage in strength from compression steel
- Main advantage of compression steel is in increasing ductility and reducing long term deflections

Analysis Problem-3

Analysis of a rectangular section 250 x 600 mm with six 28-mm bars in two layers as tension steel and two 20-mm bars as compression reinforcement. Net spacing between steel layers is 28 mm. Stirrups have 10-mm diameter

$$f_c' = 18\,MPa \qquad f_y = 420\,MPa$$



$$d' = 40 + 10 + 10 = 60\,mm$$

$$d_t = d_1 = h - 40 - 10 - 14 = 536\,mm$$

$$d_{min} = d_2 = d_1 - S_l - 28 = 536 - 56 = 480\,mm$$

$$d = \frac{d_1 + d_2}{2} = 508\,mm$$

$$A_s = 6\Phi 28 = 3694.5\,mm^2$$

$$A_s' = 2\Phi 20 = 628.3\,mm^2$$

Solution 3

Assume compression steel has yielded: $\varepsilon_s' \geq \varepsilon_y \Rightarrow f_s' = f_y$

Compute stress block depth: $a = \dfrac{A_s f_y - A_s'\left(f_y - 0.85 f_c'\right)}{0.85 f_c' b}$

$$a = \frac{3694.5 \times 420 - 628.3(420 - 0.85 \times 18)}{0.85 \times 18 \times 250} = 339.19\, mm$$

N.A. depth: $c = \dfrac{a}{\beta_1} = \dfrac{339.19}{0.85} = 399.05\, mm$

Compression steel strain: $\varepsilon_s' = 0.003 \dfrac{c - d'}{c} = 0.003 \dfrac{399.05 - 60}{399.05}$

$\varepsilon_s' = 0.00255 \qquad \varepsilon_s' \geq \varepsilon_y = 0.0021 \Rightarrow$ OK compression yielding

Tension steel strain: $\varepsilon_t = \varepsilon_1 = 0.003 \dfrac{d_1 - c}{c} = 0.003 \dfrac{536 - 399.05}{399.05}$

$\varepsilon_t = 0.00103 < \varepsilon_y \Rightarrow$ Tension steel not yielding (both layers not yielding)

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Analysis of Doubly Reinforced Beam (WSD)

WEEK-10

Tension and compression steel both at yields

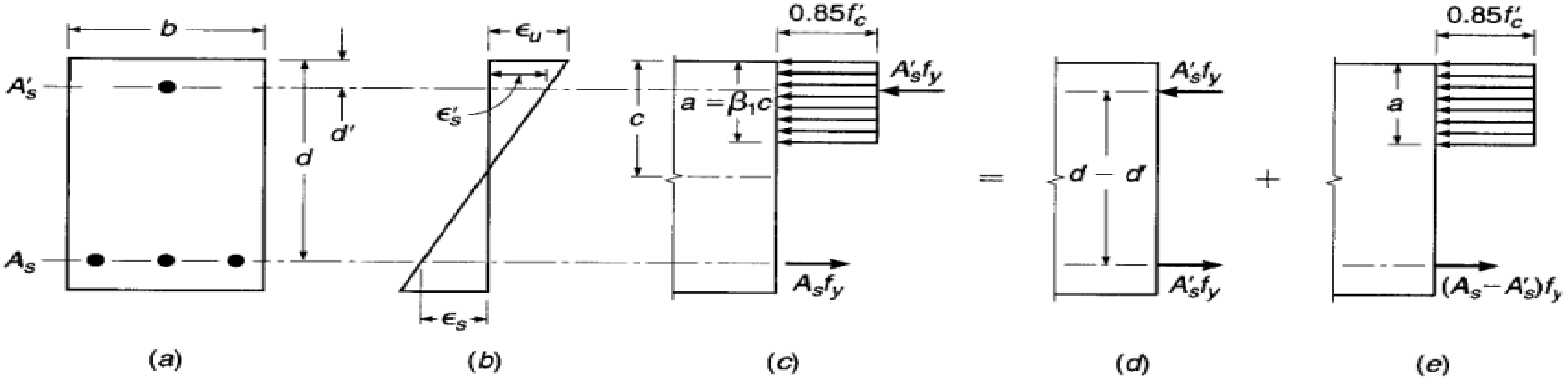


FIGURE 3.14
Doubly reinforced rectangular beam.

$$M_{n1} = A'_s f_y(d - d') \tag{3.46a}$$

as shown in Fig. 3.14*d*. The second part, M_{n2}, is the contribution of the remaining tension steel $A_s - A'_s$ acting with the compression concrete:

$$M_{n2} = (A_s - A'_s)f_y\left(d - \frac{a}{2}\right) \tag{3.46b}$$

as shown in Fig. 3.14*e*, where the depth of the stress block is

$$a = \frac{(A_s - A'_s)f_y}{0.85f'_c b} \tag{3.47a}$$

With the definitions $\rho = A_s/bd$ and $\rho' = A'_s/bd$, this can be written

$$a = \frac{(\rho - \rho')f_y d}{0.85f'_c} \tag{3.47b}$$

The total nominal resisting moment is then

$$M_n = M_{n1} + M_{n2} = A'_s f_y(d - d') + (A_s - A'_s)f_y\left(d - \frac{a}{2}\right) \tag{3.48}$$

$$\bar{\rho}_b = \rho_b + \rho' \tag{3.49}$$

where ρ_b is the balanced reinforcement ratio for the corresponding singly reinforced beam and is calculated from Eq. (3.28). The ACI Code limits the net tensile strain, not the reinforcement ratio. To provide the same margin against brittle failure as for singly reinforced beams, the maximum reinforcement ratio should be limited to

$$\bar{\rho}_{\max} = \rho_{\max} + \rho' \tag{3.50a}$$

Because $\rho_{\max}$ establishes the location of the neutral axis, the limitation in Eq. (3.50*a*) will provide acceptable net tensile strains. A check of ϵ_t is required to determine the strength reduction factor ϕ and to verify net tensile strain requirements are satisfied. Substituting $\rho_{0.005}$ for $\rho_{\max}$ in Eq. (3.50*a*) will give the maximum reinforcement ratio for $\phi = 0.90$.

$$\bar{\rho}_{0.005} = \rho_{0.005} + \rho' \tag{3.50b}$$

Compression steel below yield stress

The preceding equations, through which the fundamental analysis of doubly reinforced beams is developed clearly and concisely, are valid *only* if the compression steel has yielded when the beam reached its nominal capacity. In many cases, such as for wide, shallow beams, beams with more than the usual concrete cover over the compression bars, beams with high yield strength steel, or beams with relatively small amounts of tensile reinforcement, the compression bars will be below the yield stress at failure. It is necessary, therefore, to develop more generally applicable equations to account for the possibility that the compression reinforcement has not yielded when the doubly reinforced beam fails in flexure.

Whether or not the compression steel will have yielded at failure can be determined as follows. Referring to Fig. 3.14*b*, and taking as the limiting case $\epsilon_s' = \epsilon_y$, one obtains, from geometry,

$$\frac{c}{d'} = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} \qquad \text{or} \qquad c = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} d'$$

Summing forces in the horizontal direction (Fig. 3.14*c*) gives the *minimum* tensile reinforcement ratio $\overline{\rho}_{cy}$ that will ensure yielding of the compression steel at failure:

$$\overline{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' \tag{3.51}$$

If the *tensile reinforcement ratio* is *less than* this limiting value, the neutral axis is sufficiently high that the compression steel stress at failure is less than the yield stress. In this case, it can easily be shown on the basis of Fig. 3.14*b* and *c* that the balanced reinforcement ratio is

$$\bar{\rho}_b = \rho_b + \rho' \frac{f'_s}{f_y} \tag{3.52}$$

where

$$f'_s = E_s \epsilon'_s = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + \epsilon_y) \right] \le f_y \tag{3.53a}$$

To determine ρ_{max}, $\epsilon_t = 0.004$ is substituted for ϵ_y in Eq. (3.53*a*), giving

$$f'_s = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004) \right] \le f_y \tag{3.53b}$$

Likewise, for $\epsilon_t = 0.005$,

$$f'_s = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.005) \right] \le f_y \tag{3.53c}$$

Hence, the maximum reinforcement ratio permitted by the ACI Code is

$$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f'_s}{f_y} \tag{3.54a}$$

and the maximum reinforcement ratio for $\phi = 0.90$ is

$$\bar{\rho}_{0.005} = \rho_{0.005} + \rho' \frac{f'_s}{f_y} \tag{3.54b}$$

If the tensile reinforcement ratio is less than $\overline{\rho}_b$, as given by Eq. (3.52), and less than $\overline{\rho}_{cy}$, as given by Eq. (3.51), then the tensile steel is at the yield stress at failure but the compression steel is not, and new equations must be developed for compression steel stress and flexural strength. The compression steel stress can be expressed in terms of the still-unknown neutral axis depth as

$$f'_s = \epsilon_u E_s \frac{c - d'}{c} \tag{3.55}$$

Consideration of horizontal force equilibrium (Fig. 3.14*c* with compression steel stress equal to f'_s) then gives

$$A_s f_y = 0.85\beta_1 f'_c bc + A'_s \epsilon_u E_s \frac{c - d'}{c} \tag{3.56}$$

This is a quadratic equation in c, the only unknown, and is easily solved for c. The nominal flexural strength is found using the value of f'_s from Eq. (3.55), and $a = \beta_1 c$ in the expression

$$M_n = 0.85 f'_c ab\left(d - \frac{a}{2}\right) + A'_s f'_s (d - d') \tag{3.57}$$

TABLE 3.2
Minimum beam depths for compression reinforcement to yield

	$\epsilon_t = 0.004$		$\epsilon_t = 0.005$	
f_y, psi	Maximum d'/d	Minimum d for $d' = 2.5$ in., in.	Maximum d'/d	Minimum d for $d' = 2.5$ in., in.
40,000	0.23	10.8	0.20	12.3
60,000	0.13	18.8	0.12	21.5
75,000	0.06	42.7	0.05	48.8

Example 3.12

Flexural strength of a given member. A rectangular beam, shown in Fig. 3.15, has a width of 12 in. and an effective depth to the centroid of the tension reinforcement of 24 in. The tension reinforcement consists of six No. 10 (No. 32) bars in two rows. Compression reinforcement consisting of two No. 8 (No. 25) bars is placed 2.5 in. from the compression face of the beam. If $f_y = 60{,}000$ psi and $f'_c = 5000$ psi, what is the design moment capacity of the beam?

Solution. The steel areas and ratios are

$$A_s = 7.62 \text{ in}^2 \qquad \rho = \frac{7.62}{12 \times 24} = 0.0265$$

$$A'_s = 1.58 \text{ in}^2 \qquad \rho' = \frac{1.58}{12 \times 24} = 0.0055$$

Check the beam first as a singly reinforced beam to see if the compression bars can be disregarded,

$$\rho_{max} = 0.0243 \qquad \text{from Table A.4 or Eq. (3.30}c\text{)}$$

The actual $\rho = 0.0265$ is larger than ρ_{max}, so the beam must be analyzed as doubly reinforced. From Eq. (3.51), with $\beta_1 = 0.80$,

$$\bar{\rho}_{cy} = 0.85 \times 0.80 \times \frac{5}{60} \times \frac{2.5}{24} \times \frac{0.003}{0.003 - 0.00207} + 0.0055 = 0.0245$$

The tensile reinforcement ratio is greater than this, so the compression bars will yield when the beam fails. The maximum reinforcement ratio thus can be found from Eq. (3.50),

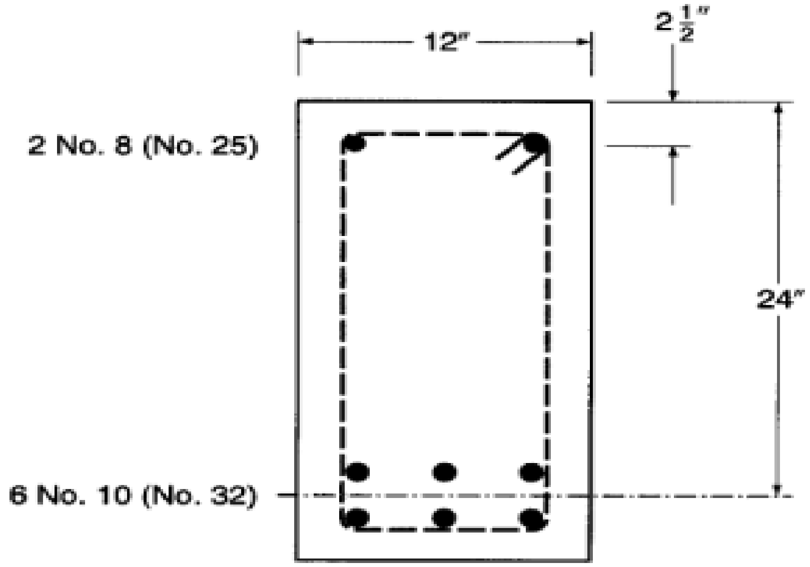
$$\overline{\rho}_{max} = 0.0243 + 0.0055 = 0.0298$$

The actual tensile reinforcement ratio is below the maximum value, as required. Then, from Eq. (3.47*a*),

$$a = \frac{(7.62 - 1.58)60}{0.85 \times 5 \times 12} = 7.11 \text{ in.}$$

$$c = a/\beta_1 = \frac{7.11}{0.80} = 8.89 \text{ in.}$$

$$\epsilon_t = 0.003\left(\frac{24 - 8.89}{8.89}\right) = 0.0051$$



and

$$\phi = 0.90$$

and from Eq. (3.48),

$$M_n = 1.58 \times 60(24 - 2.5) + 6.04 \times 60\left(24 - \frac{7.11}{2}\right) = 9450 \text{ in-kips}$$

The design strength is

$$\phi M_n = 0.90 \times 9450 = 8500 \text{ in-kips}$$

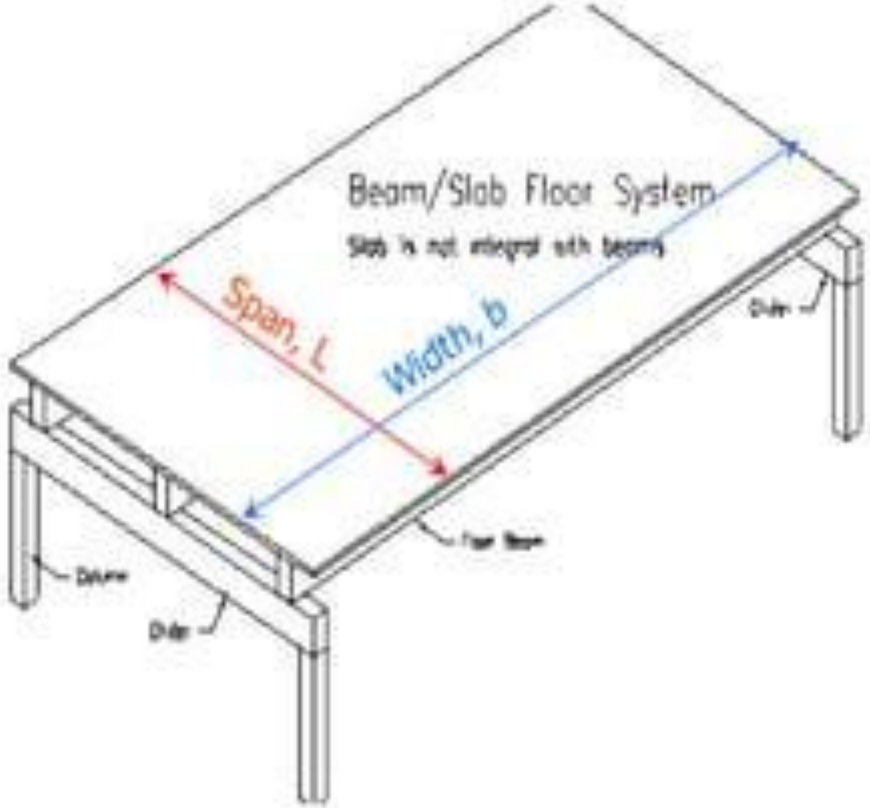
Design of One-Way Slab

WEEK 11-12

SLAB

Slabs are an important structural component where prestressing is applied.

Slabs are used to provide flat or useful surfaces.



TYPES OF SLAB

ONE WAY SLAB

One-way slabs are those slabs with an aspect ratio in plan of 2:1 or greater, in which bending is primarily about the long axis.

So, the slab is one way where $L/B \geq 2$.

TYPES OF ONE WAY SLAB

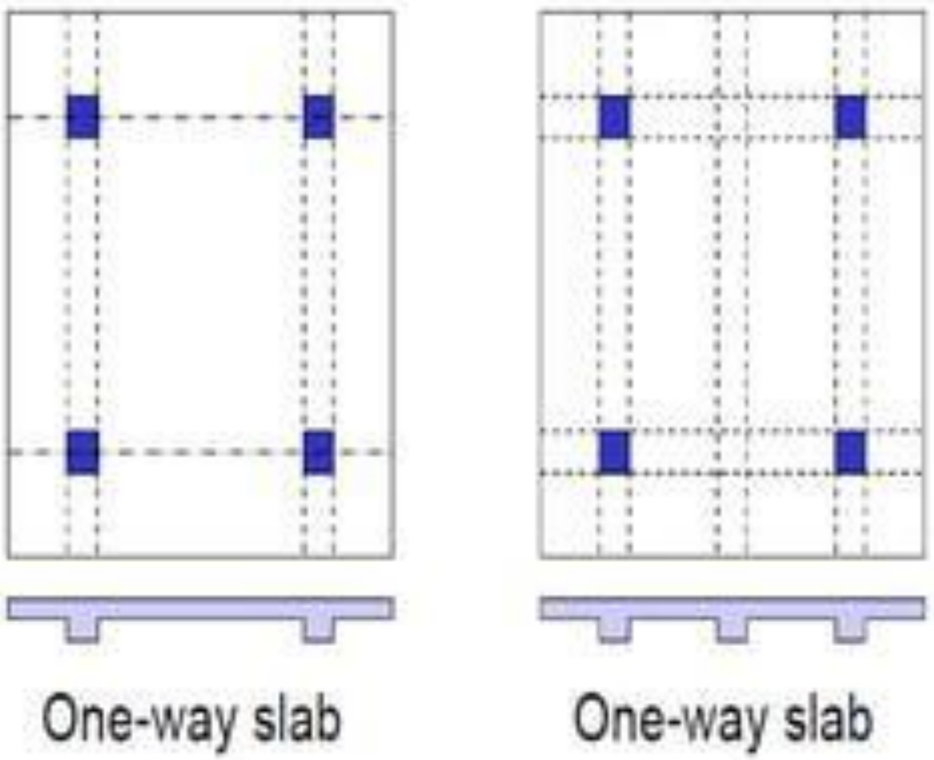
One way slab may be...

- SOLID
- HOLLOW
- RIBBED

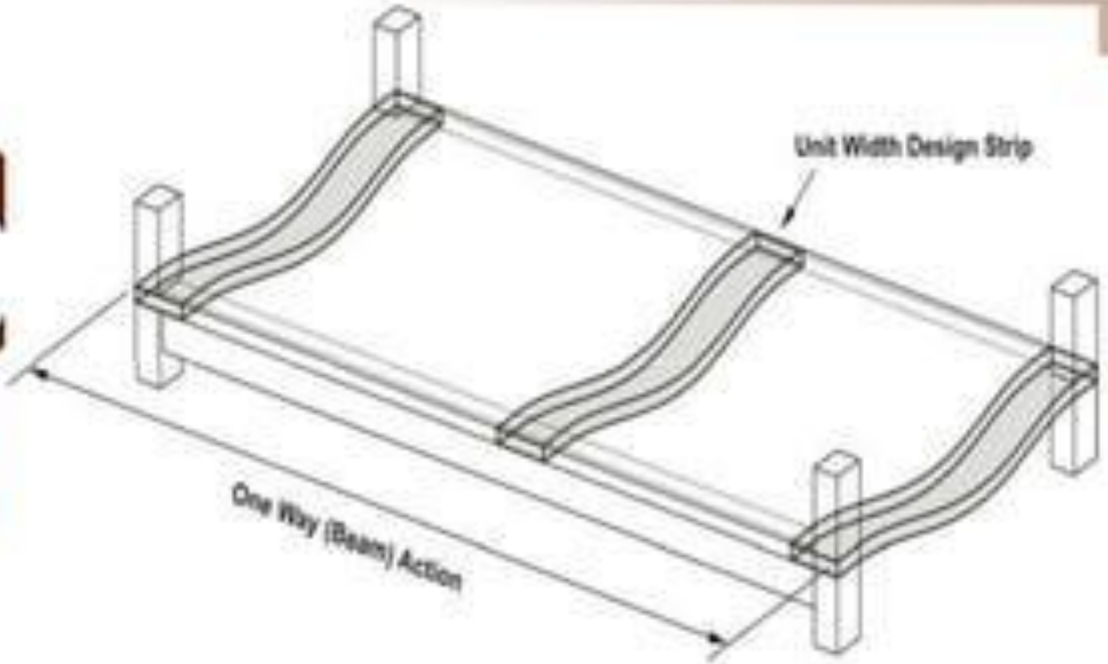
APPLICATION OF L/B RATIO

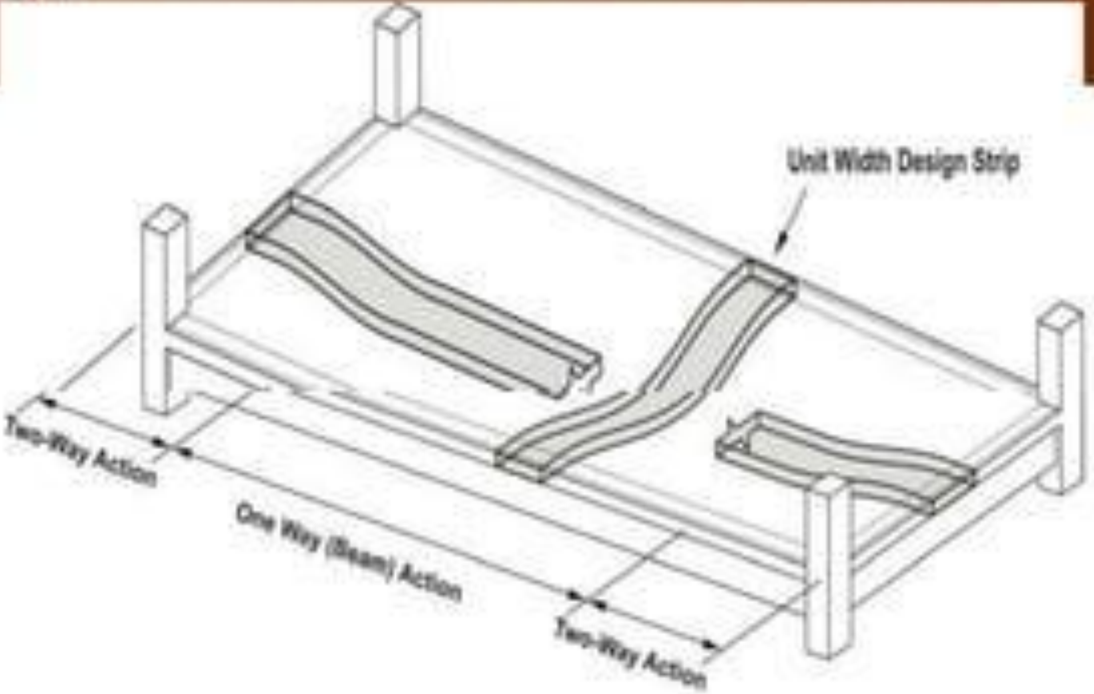
In first figure slab is supported on two opposite sides only. In this case the structural action of the slab is essentially one way.

In second figure there are beams on all four sides with a intermediate beam. Now if length to width ratio is 2 or greater, slab is one way even though supports are provided on all sides.



LOADING OF ONE WAY SLAB





When slabs are supported on two opposite sides only loads being carried by the slab in the direction perpendicular to the supporting beams.

When supports are provided on all sides most of the load is carried in the short direction to the supporting beams and one way action is obtained.

DESIGN & ANALYSIS

For analysis there is a term as….

"ONE WAY SLAB IS A SET OF A RECTANGULAR BEAMS SIDE BY SIDE"

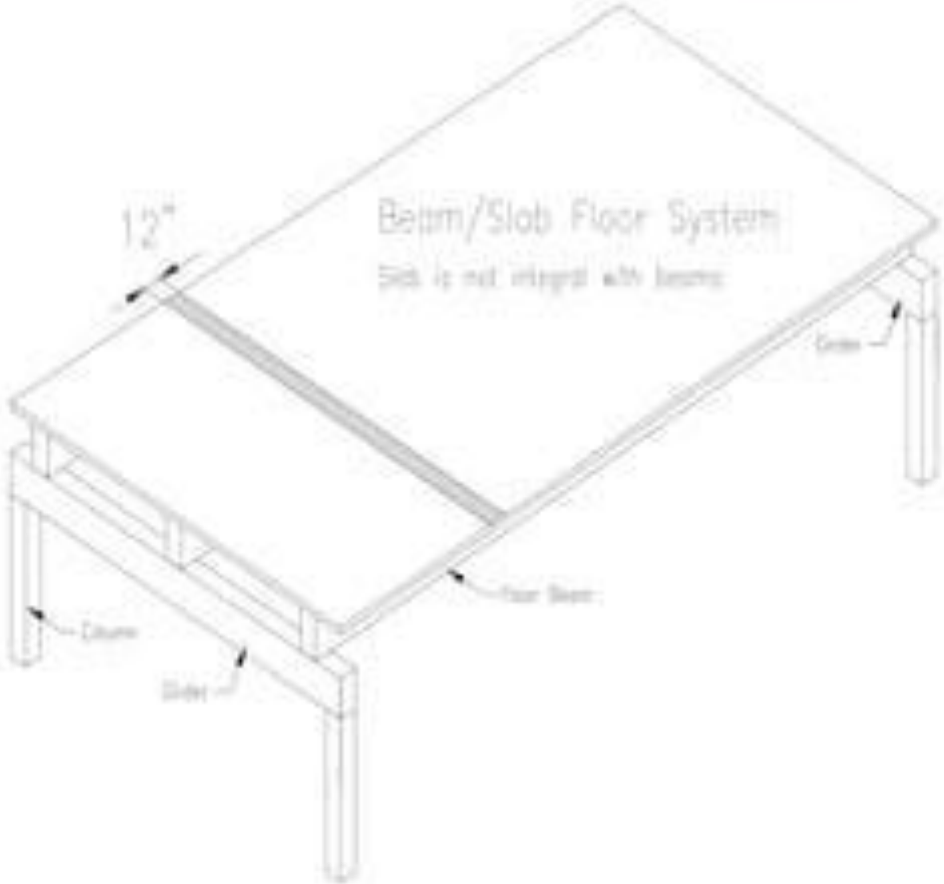
But How ???

Lets find it…..

For purpose of analysis and design a unit strip of such a slab is cut out , which may be considered as a rectangular beam of unit width (say 1ft or 1m) with a depth ‘h’ equal to the thickness of the slab and a span ‘l’ equal to the distance between supported edges.

The strip can be analyzed by the methods that were used for rectangular beams.

So that term is clear.



ACI CODE SPECIFICATIONS

For design purposes there are some ACI code specifications.

Such as…..

MINIMUM SLAB THICKNESS

To control deflection, *ACI Code 9.5.2.1 specifies minimum thickness values for one-way* solid slabs.

Element	Simply supported	One end continuous	Both ends continuous	Cantilever
One-way solid slabs	$l/20$	$l/24$	$l/28$	$l/10$

MINIMUM CONCRETE COVER

According to *ACI Code 7.7.1, the following minimum concrete cover is to be provided:*

a. Concrete not exposed to weather or in contact with ground:

- Larger than *Ø 36 mm bar --4 cm*
- *Ø 36 mm and smaller bars --2 cm*

b. Concrete exposed to weather or in contact with ground:

- *Ø 19 mm and larger bars---5 cm*

Ø 16 mm and smaller bars --4 cm

c. Concrete cast against and permanently exposed to earth ----------*7.5 cm*

SPAN

According to ACI code 8.7.1

If the slab rests freely on its supports the span length may be taken equal to the clear span plus the depth of the slab but need not exceed the distance between centers of supports .

BAR SPACING

The lateral spacing of the flexural bars should not exceed 3 times the thickness h or 18 inch according to ACI code 7.6.5

The lateral spacing of temperature and shrinkage reinforcement should not be placed farther apart than 5 times the slab thickness or 18 inch according to ACI code 7.12.2

MAXIMAM REINFORCEMENT RATIO

REINFORCEMENT RATIO : Reinforcement ratio is the ratio of reinforcement area to gross concrete area based on total depth of slab.

One-way solid slabs are designed as rectangular sections subjected to shear and moment. Thus, the maximum reinforcement ratio corresponds to a net tensile stain in the reinforcement, *€t* of 0.004

MINIMAM REINFORCEMENT RATIO

For temperature and shrinkage reinforcement :

According to *ACI Code 7.12.2.1*

Slabs with Grade 40 or 50 deformed bars............ 0.0020

Slabs with Grade 60 deformed bars 0.0018

Slabs where reinforcement with yield strength Exceeding 60000 psi .... $\frac{0.0018 * 60000}{fy}$

For flexural reinforcement :

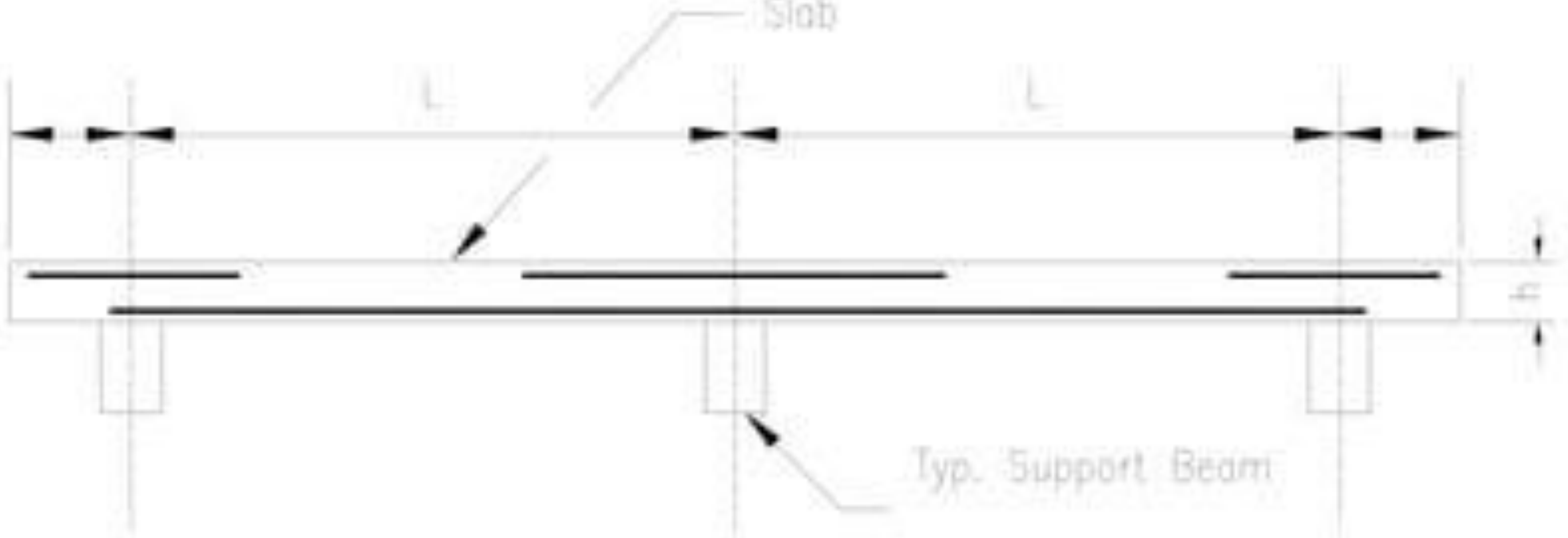
According to *ACI Code 10.5.4,*

the minimum flexural reinforcement is not to be less than the shrinkage reinforcement, or *0.0018*

EXAMPLE PROBLEM

A reinforced concrete slab is built integrally with its supports and consists of equal span of 15 ft. The service live load is 100 psf and 4000 psi concrete is specified for use with steel with a yield stress equal to 60000 psi. Design the slab following the provisions of the ACI code.

BEAM PROFILE



Design variables: Thickness (h) and Reinforcing.

THICKNESS ESTIMATION

For being both ends continuous minimum slab thickness = L/28=(15*120)/28=6.43 in.

Let a trial thickness of 6.50 in.

DETERMINING LOADS

- Consider only a 1 ft width of beam .
- Dead load = 150*6.50/12=81 psf
- Live load = 100 psf
- Factored DL and LL =(81+1.2+100*1.6)

  =257 psf

DETERMINING MAXIMUM MOMENTS

- Factored moments at critical sections by ACI code :
- At interior support : $-M=1/9 * 0.257 * 15^2 = 6.43$ k-ft
- At midspan : $+M=1/14 * 0.257 * 15^2 = 4.13$ k-ft
- At exterior support : $-M=1/24 * 0.257 * 15^2 = 2.41$ k-ft
- Mmax = 6.43 k-ft

MINIMUM EFFECTIVE DEPTH

$$\rho = 0.85\beta \frac{fc'}{fy} \frac{\varepsilon u}{\varepsilon u + \varepsilon t}$$

=0.85*0.85*4/60*0.003/(0.003+0.004)

= 0.021

$$\text{Now, } d = \sqrt{\frac{Mmax}{\text{Ø}\rho fyb(1-0.59\rho fy/fc')}}$$

= 2.64 in

CHECKING AVAILABILITY OF THICKNESS

As 'd' is less than effective depth of (6.50-1.00)=5.50 in, the thickness of 6.50 in can be adopted.

REINFORCEMENT CALCULATION

Let, a=1 in

At Interior Support :

$$A_s = \frac{Mu}{\phi fy(d-\frac{a}{2})} = \frac{6.43*12}{0.90*60*5.00} = 0.29\ in^2$$

Checking the assumed depth 'a' by

$a = A_s f_y / 0.85 f_c' b = (0.29*60)/(0.85*4*12) = 0.43$ in

For , a=0.43 in $A_s = 0.27\ in^2$

Similarly at Midspan :

$A_s = (4.13*12)/(0.90*60*5.29) = 0.17\ in^2$

At Exterior Support :

$A_s = (2.41*12)/(0.90*60*5.29) = 0.10\ in^2$

MINIMUM REINFORCEMENT

A_s= 0.0018*12*6.50=0.14 in^2

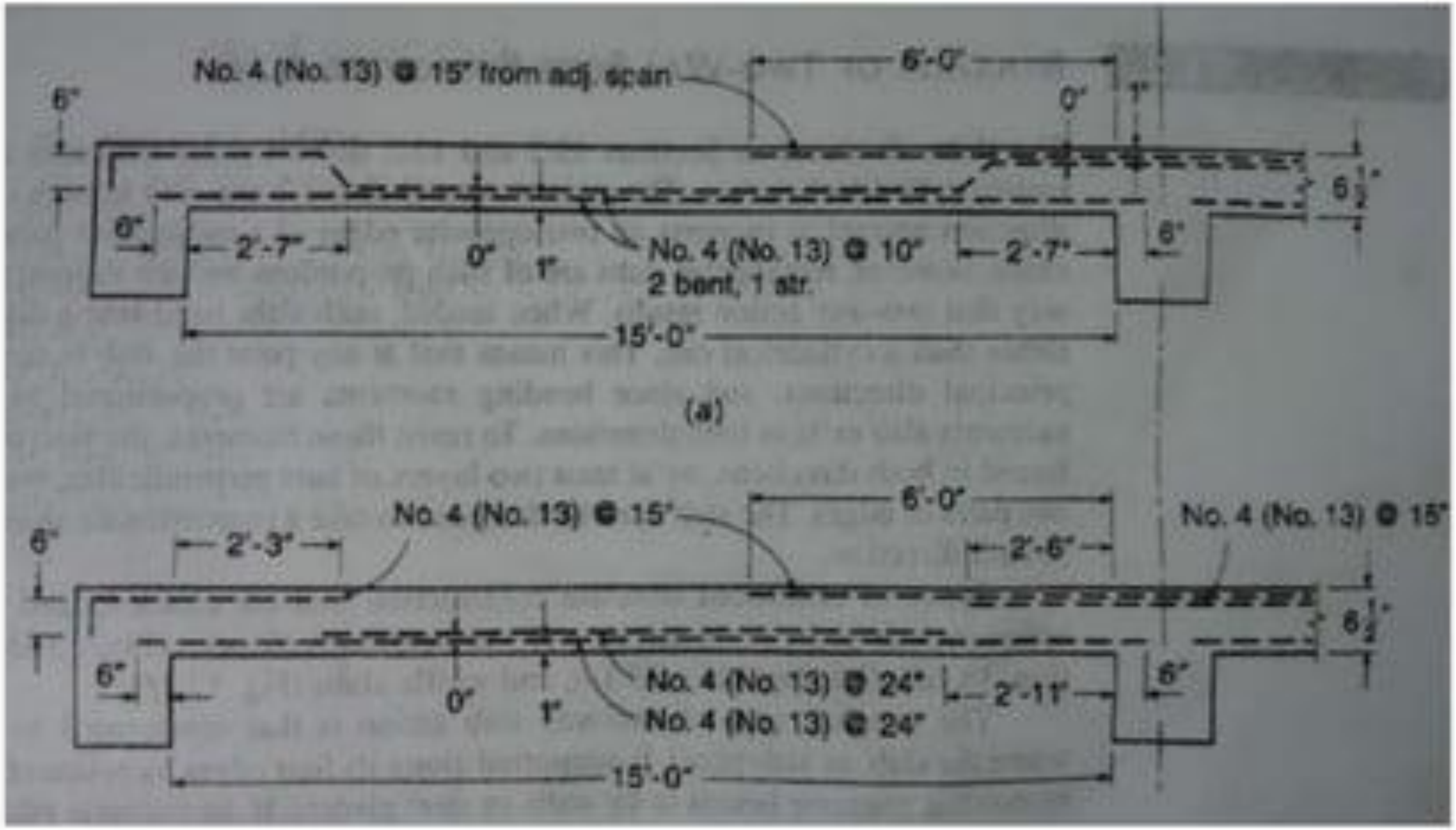
So we have to provide this amount of reinforcement where A_s is less than 0.14 in^2

SHRINKAGE REINFORCEMENT

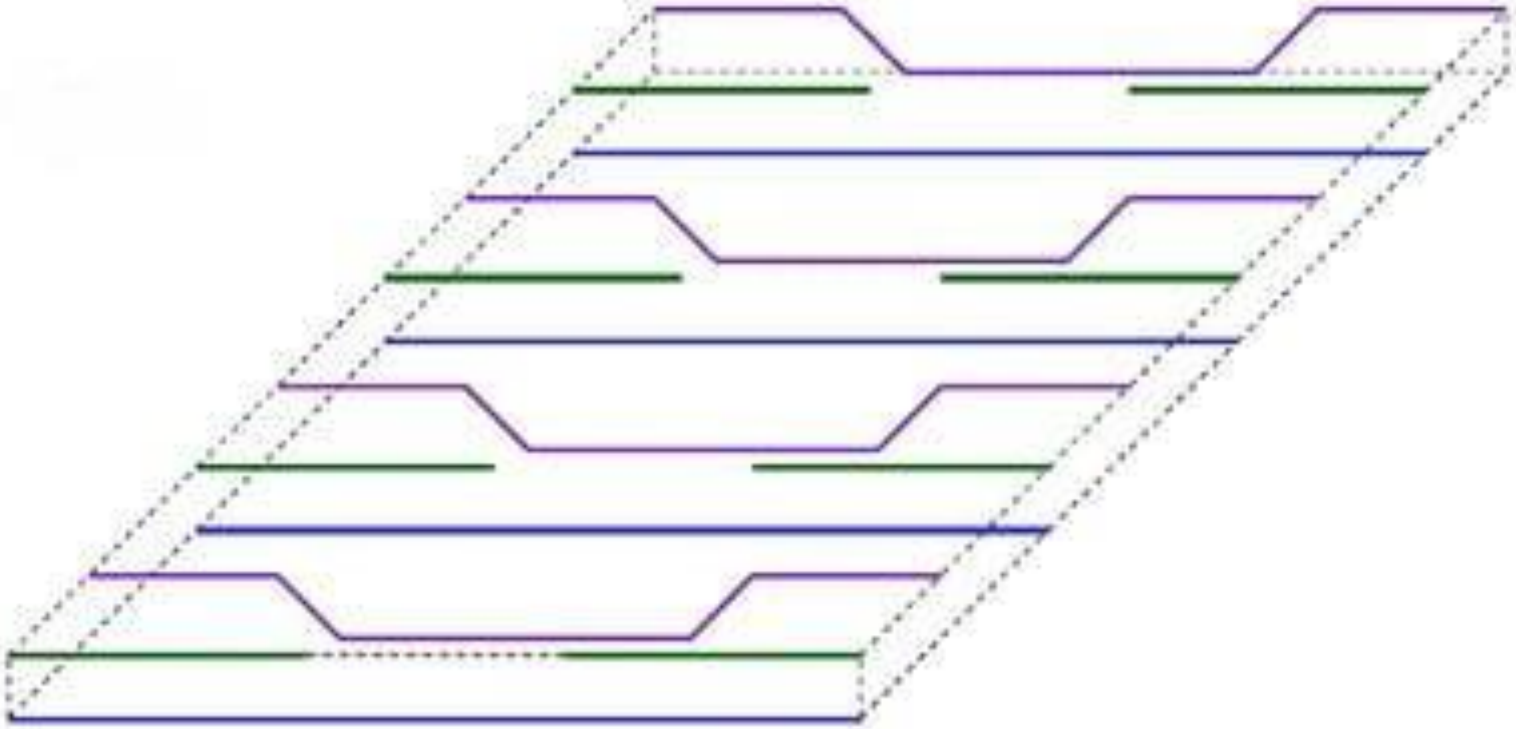
Minimum reinforcement for shrinkage and temperature is

$A_s = 0.0018*12*6.50=0.14\ in^2$

FINAL DESIGN



LAYOUT OF ONE WAY SLAB



APPLICATION OF ONE WAY SLAB

- Provides useful flat surface
- One way slab may be used when there is architectural limitations
- It is the simplest form of slab design
- Main reinforcement placing is one way, so there is a little congestion than two way slab

Shear Design Procedure according to ACI

WEEK-13

Shear Analysis and Design according to SBC / ACI Codes

- SBC, ACI and other codes neglect aggregate interlock and dowel action
- Shear force is resisted by compression concrete and by steel stirrups only
- Nominal shear strength is therefore:

$$V_n = V_c + V_s$$

V_c : Concrete shear strength

V_s : Stirrups shear strength

Steel stirrups may be inclined along diagonal tension but are usually vertical

Shear Analysis and Design according to SBC / ACI Codes

Nominal shear strength is provided by concrete and stirrups only
Design shear strength must be equal to or greater than ultimate shear

$$\phi V_n \geq V_u \quad \text{with} \quad V_n = V_c + V_s$$

V_c : Concrete shear strength

V_s : Stirrups shear strength

With $\underline{\underline{\phi = 0.75}}$ in shear

- Concrete may provide enough strength to resist ultimate shear but SBC / ACI require stirrups if:

$$V_n > \frac{V_c}{2} \Leftrightarrow \frac{V_u}{\phi} > \frac{V_c}{2} \Leftrightarrow V_u > \frac{\phi V_c}{2}$$

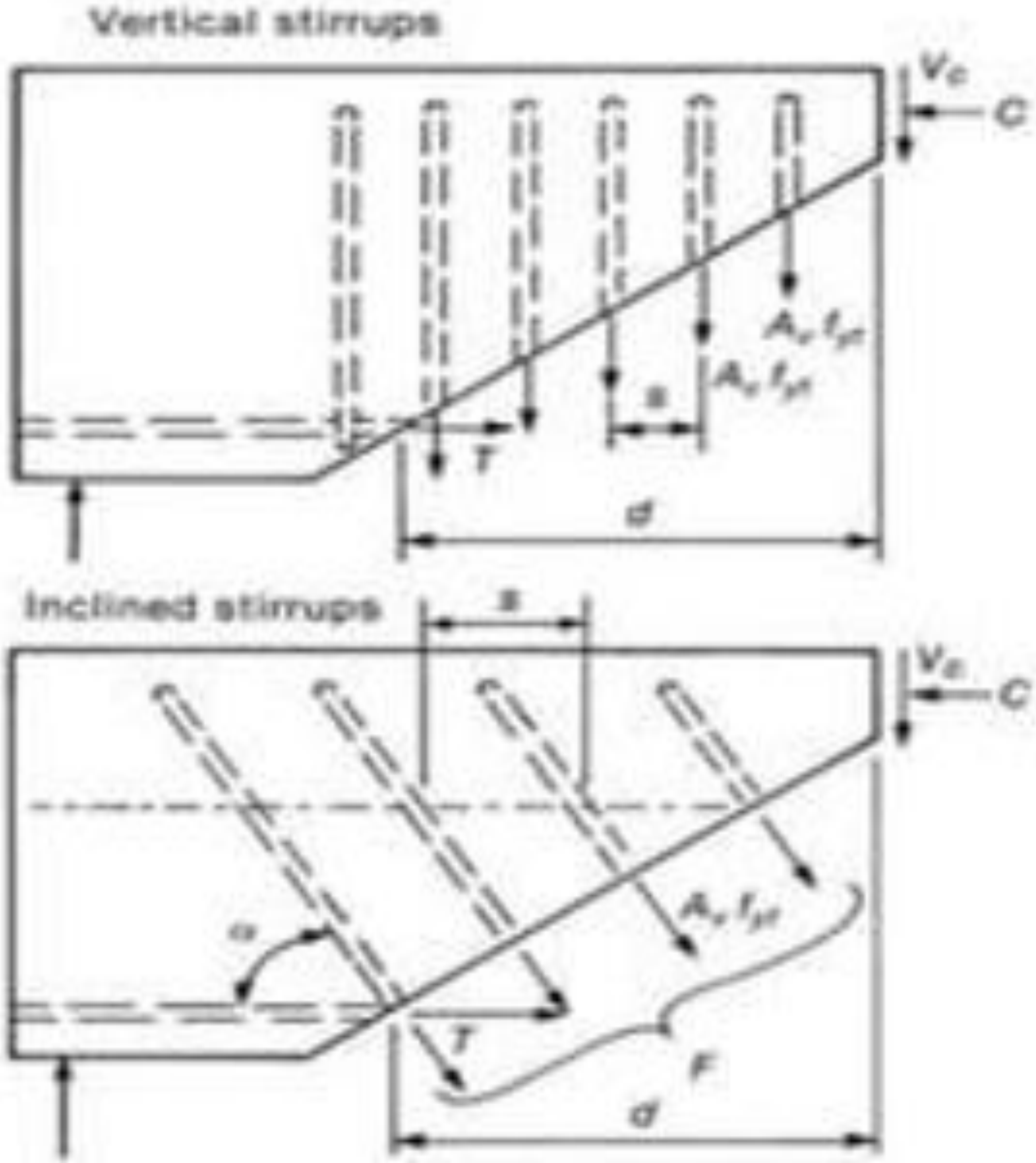
Shear Analysis and Design per SBC / ACI

- Assuming a 45^0 inclined crack, the number of vertical stirrups crossed by the crack is $n_s = d/s$ where s is the stirrup spacing
- Assuming that they have yielded the stirrups shear strength is :

Vertical stirrups: $V_s = n_s A_v f_y = \dfrac{A_v f_y d}{s}$

Inclined stirrups: $V_s = A_v f_y (\sin\alpha + \cos\alpha)\dfrac{d}{s}$

A_v : Stirrup cross-section area



Shear Analysis and Design per SBC / ACI

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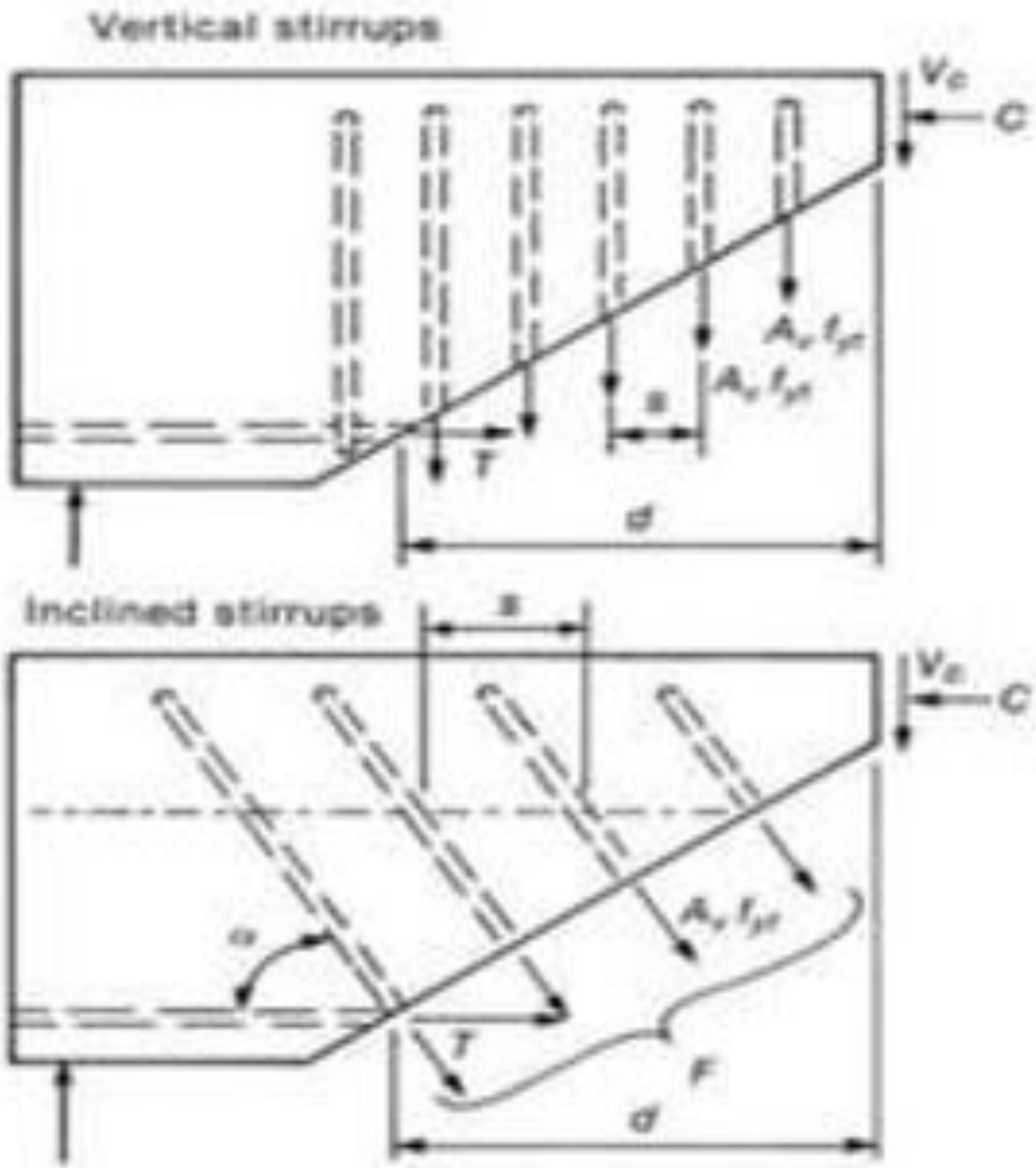
A_v : Stirrup cross - section area



Stirrup Section Area

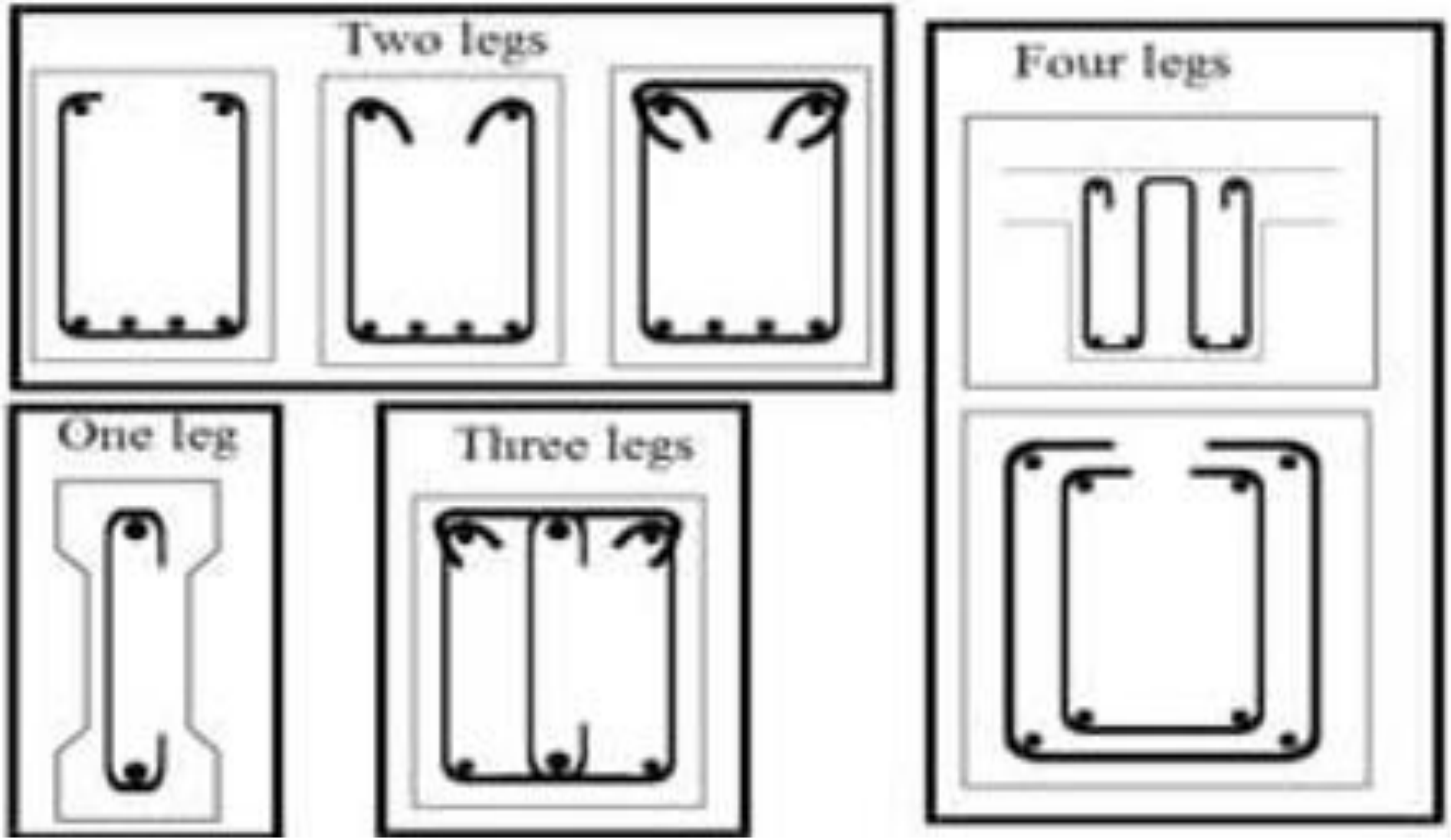
- A stirrup has at least one leg but usually two legs or more.

$$V_s = \frac{A_v f_y d}{s}$$

Stirrup with n legs :

$$A_v = n\pi \frac{d_s^{\,2}}{4}$$

d_s : Stirrup diameter



Section Adequacy for Shear

- Before performing design, the section must first be checked whether it is sufficient to resist shear according to SBC / ACI.
- The section is insufficient to resist shear and must be increased if :

$$V_s > \frac{2}{3}\sqrt{f_c'}\,b_w d \Leftrightarrow \frac{V_u}{\phi} - V_c > \frac{2}{3}\sqrt{f_c'}\,b_w d$$

but $\quad V_c = \frac{\sqrt{f_c'}}{6} b_w d$

$\Rightarrow$ Insufficient section for shear if : $V_u > 5\phi V_c$

In terms of average shear stress, the condition is :

Insufficient section for shear if : $v_u = \frac{V_u}{b_w d} > 5\phi \frac{\sqrt{f_c'}}{6}$

Beam Shear Design
Required Vertical Stirrup Spacing

- SBC / ACI specify that stirrups are required in beams if :

$$V_n > \frac{V_c}{2} \Leftrightarrow V_u > \frac{\phi V_c}{2}$$

$\phi V_n \geq V_u$ with $V_n = V_c + V_s$ Optimal design $\Leftrightarrow \phi V_n = V_u$

$$\Rightarrow \phi(V_c + V_s) = V_u \Rightarrow V_s = \frac{V_u}{\phi} - V_c \quad \Rightarrow \frac{A_v f_y d}{s} = \frac{V_u}{\phi} - V_c$$

$\Rightarrow$ Required stirrup spacing : $s = \dfrac{A_v f_y d}{\dfrac{V_u}{\phi} - V_c}$

Note: if $\left(V_s = \frac{V_u}{\phi} - V_c\right) \leq 0 \Rightarrow$ Use minimum steel requirement

Minimum Web Reinforcement and Maximum Stirrup Spacing

SBC / ACI minimum web steel area : $A_{v,\min} = \mathrm{Max}\left(\frac{\sqrt{f_c'}}{16}, \frac{1}{3}\right)\frac{b_w s}{f_y}$

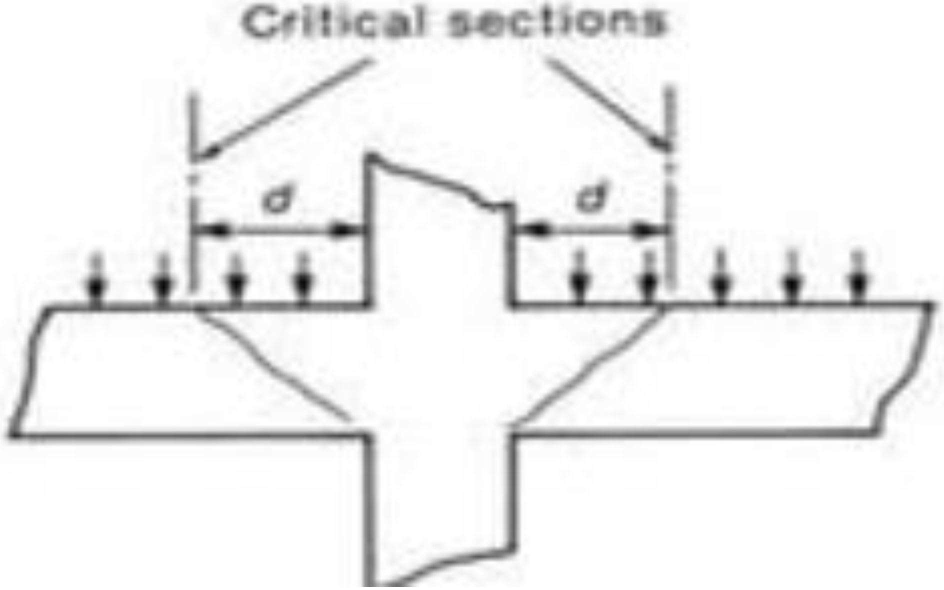
SBC / ACI maximum stirrup spacing (geometry considerations) :

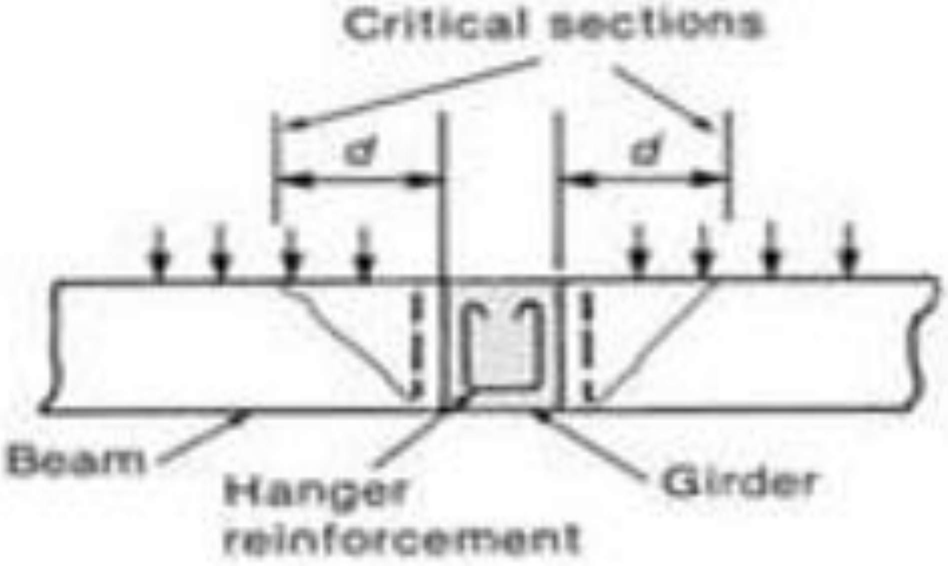
$$\begin{cases} \text{If} \quad V_s \le 2V_c & s_{\max} = \mathrm{Min}(0.5d\ ,\ 600\,mm) \ \text{Case (a)} \\ \text{If} \quad V_s > 2V_c & s_{\max} = \mathrm{Min}(0.25d\ ,\ 300\,mm) \ \text{Case (b)} \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{If} \quad V_u \le 3\phi V_c & s_{\max} = \mathrm{Min}(0.5d\ ,\ 600\,mm) \ \text{Case (a)} \\ \text{If} \quad V_u > 3\phi V_c & s_{\max} = \mathrm{Min}(0.25d\ ,\ 300\,mm) \ \text{Case (b)} \end{cases}$$

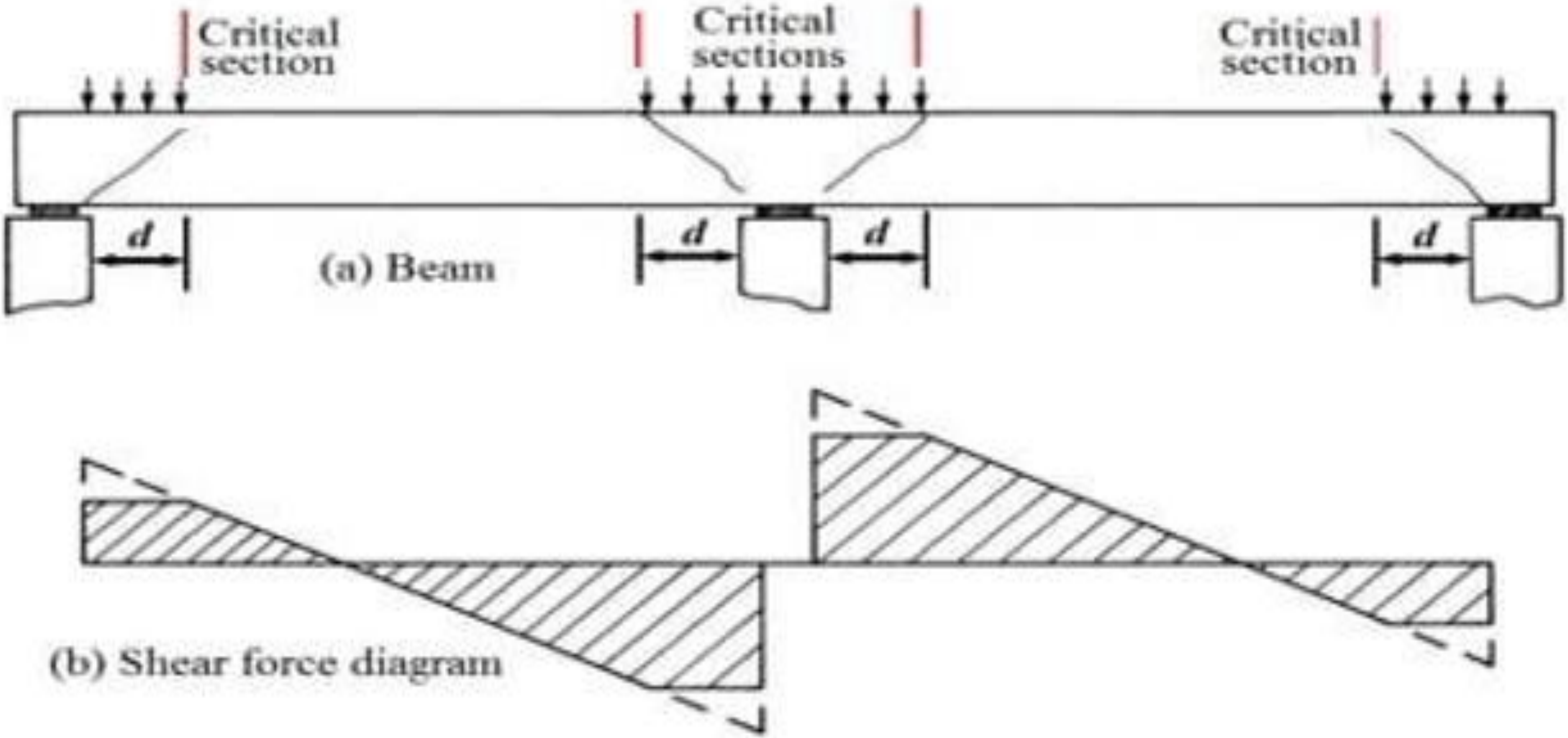
Critical Shear Sections

- Load transfer between beams and supports is performed through conic shaped zones with 45^0 angles.
- Shear failures occur at critical sections located at a distance *d* from the face of the support, where *d* is the depth of tension steel
- Ultimate shear force considered for design is computed at the critical section



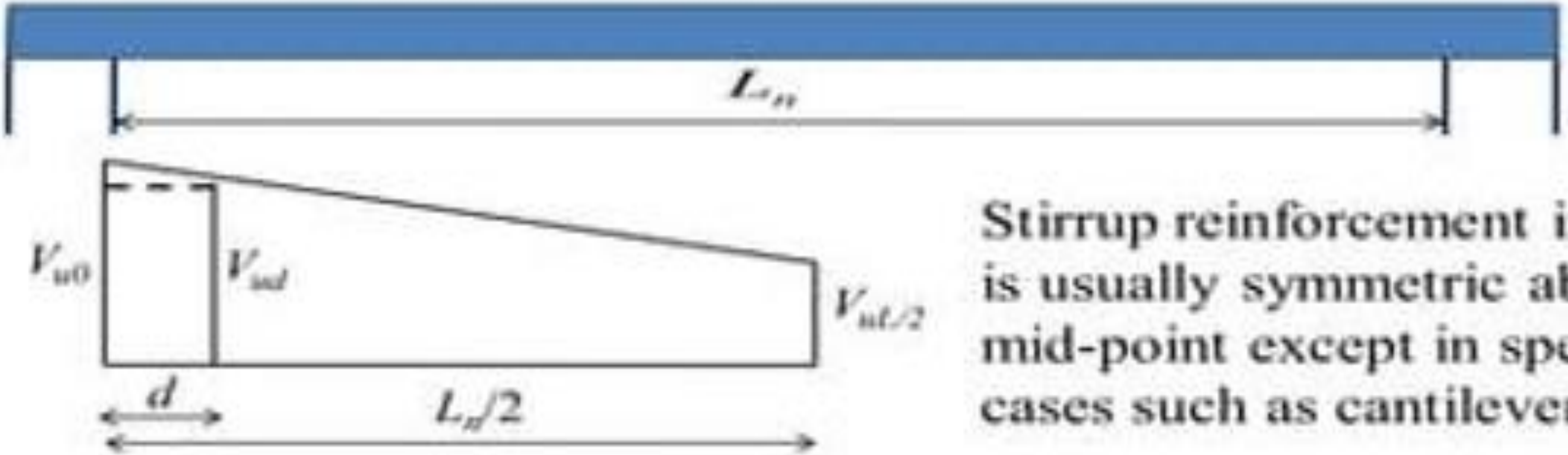


Critical Shear Sections



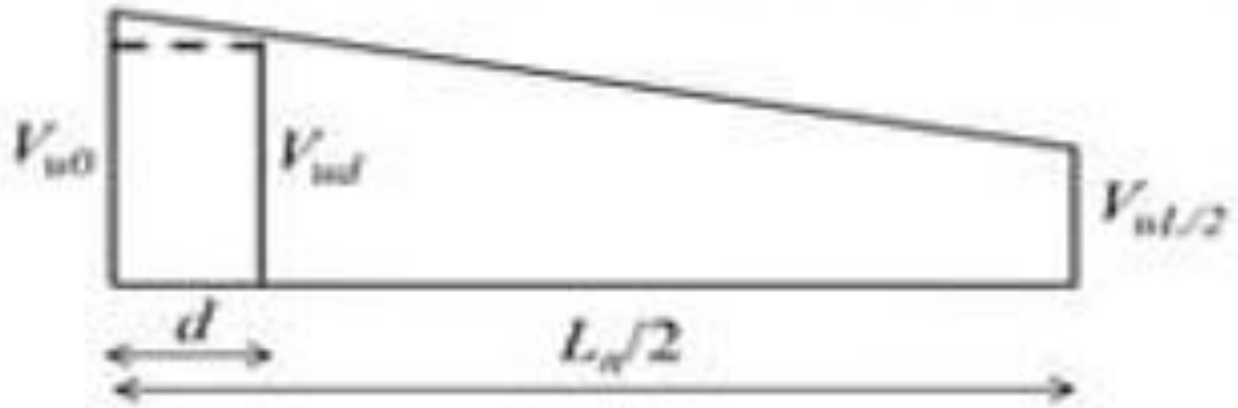
Critical Shear Section for Design

- Ultimate shear used for design is computed at the critical section at a distance d from the support face (d = Tension steel depth)
- It is obtained from support and mid-span values using shear force envelope diagram.
- Most modern structural analysis methods use clear length L_n (clear distance between support faces)



Stirrup reinforcement in beams is usually symmetric about mid-point except in special cases such as cantilevers

Critical Shear Section for Design



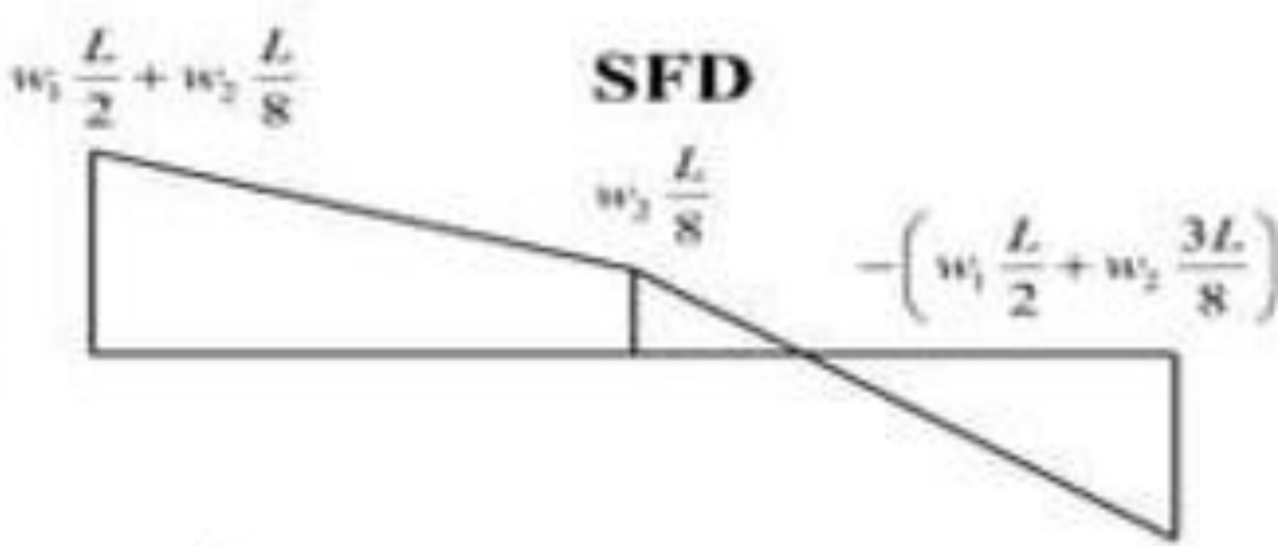
- For a simply supported beam, the ultimate shear force values at the support and mid-span are : $V_{u0} = \frac{w_u L_n}{2}$ $V_{uL/2} = \frac{w_{Lu} L_n}{8}$

w_u : Total factored (ultimate) uniform load ($= 1.4w_D + 1.7w_L$)
w_{Lu} : Factored live load ($= 1.7w_L$)

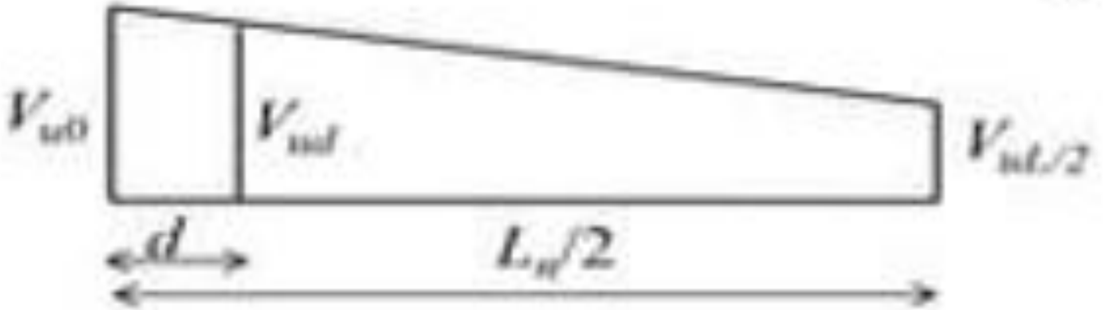
Shear force at support is obtained with ultimate uniform load applied on all the span
The mid-span value is obtained by applying factored live load on half the span only (along with factored dead load on all span).

Critical Shear Section for Design

Mid-span shear value



From similar triangles we obtain the critical shear value at distance d :



$$V_{ud}=V_{u0}-\frac{2d}{L_n}\left(V_{u0}-V_{uL/2}\right)$$

with $V_{u0}=\frac{w_u L_n}{2}$, $V_{uL/2}=\frac{w_{Lu} L_n}{8}$

Shear Design Problem 1

- A simply supported beam is subjected to uniform loading composed of dead load (including self weight) of 27.0 kN/m and live load of 17.5 kN/m.
- The beam clear span length is 9.6 m and the section dimensions are 300 x 600 mm.
- Steel depth is $d = 540$ mm
- Design the beam for shear using 10 mm stirrups and the following material data :

$$f_c^{'} = 25\,MPa \qquad f_y = 420\,MPa$$

Solution 1

The ultimate load is :

$w_u = 1.4 \times 27.0 + 1.7 \times 17.5 = 37.8 + 29.75 = 67.55$ kN/m

Factored live load is : $w_{Lu} = 1.7 \times 17.5 = 29.75$ kN/m

Ultimate shear force at support and mid-span as well as value at the critical section (at a distance d from the support) are:

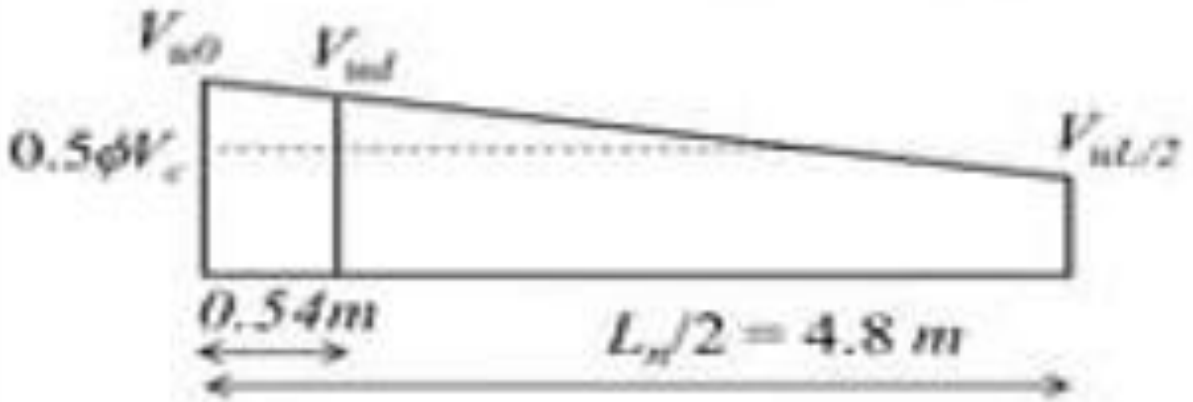
$$V_{u0} = w_u \frac{L_n}{2} = 67.55 \frac{9.6}{2} = 324.24\,kN$$

$$V_{uL/2} = w_{Lu} \frac{L_n}{8} = 29.75 \frac{9.6}{8} = 35.7\,kN$$

$$V_{ud} = V_{u0} - \frac{2d}{L_n}(V_{u0} - V_{uL/2})$$

$$V_{ud} = 324.24 - \frac{2 \times 0.54}{9.6}(324.24 - 35.7) = 291.78\,kN$$

Solution 1 – Cont.



$0.5\phi V_c = 50.62\,kN$

$V_{u0} = 324.24\,kN$

$V_{uL/2} = 35.7\,kN$

$V_{ud} = 291.78\,kN$

V_{ud} is used for design

Concrete nominal shear strength is :

$$V_c = \frac{\sqrt{f_c}}{6} b_w d = \frac{\sqrt{25}}{6} 300 \times 540 = 135000\,N = 135.0\,kN$$

Section adequacy check :

$5\phi V_c = 5 \times 0.75 \times 135 = 506.25\,kN > V_u = V_{ud} = 291.78\,kN \Rightarrow$ Section OK

Stirrup requirement : $0.5\phi V_c = 50.625\,kN < V_{ud} \Rightarrow$ Stirrups are required

Use 3 leg stirrups: $n = 3 \Rightarrow A_v = \dfrac{n\pi d_s^{\,2}}{4} = \dfrac{3\pi 10^2}{4} = 235.6\,mm^2$

Solution 1 – Cont.

$$\text{Maximum geometry spacing}: V_{ud} = 291.78 < 3\phi V_c = 303.75\,kN$$

$$\Rightarrow s^1_{max} = \text{Min}(0.5d\ ,\ 600\,mm) = 270.0\,mm \quad \text{Case (a)}$$

$$\text{Minimum steel spacing}: s^2_{max} = \text{Min}\left(\frac{16.0}{\sqrt{f_c'}}, 3.0\right)\frac{A_v f_y}{b_w}$$

$$\Rightarrow s^2_{max} = \text{Min}\left(\frac{16.0}{\sqrt{25}}, 3.0\right)\frac{235.6 \times 420}{300} = 989.5\,mm$$

$$\text{Required stirrup spacing}: s^3_{max} = \frac{A_v f_y d}{\frac{V_{ud}}{\phi} - V_c}$$

$$\Rightarrow s^3_{max} = \frac{235.6 \times 420 \times 540}{\left(\frac{291.78}{0.75} - 135.0\right) \times 1000} = 210.3\,mm$$

Solution 1 – Cont.

Maximum spacing requirement summary :

Geometry maximum spacing : $s_{max}^{1} = 270.0\,mm$

Minimum steel spacing : $s_{max}^{2} = 989.5\,mm$

Required stirrup spacing : $s_{max}^{3} = 210.3\,mm$

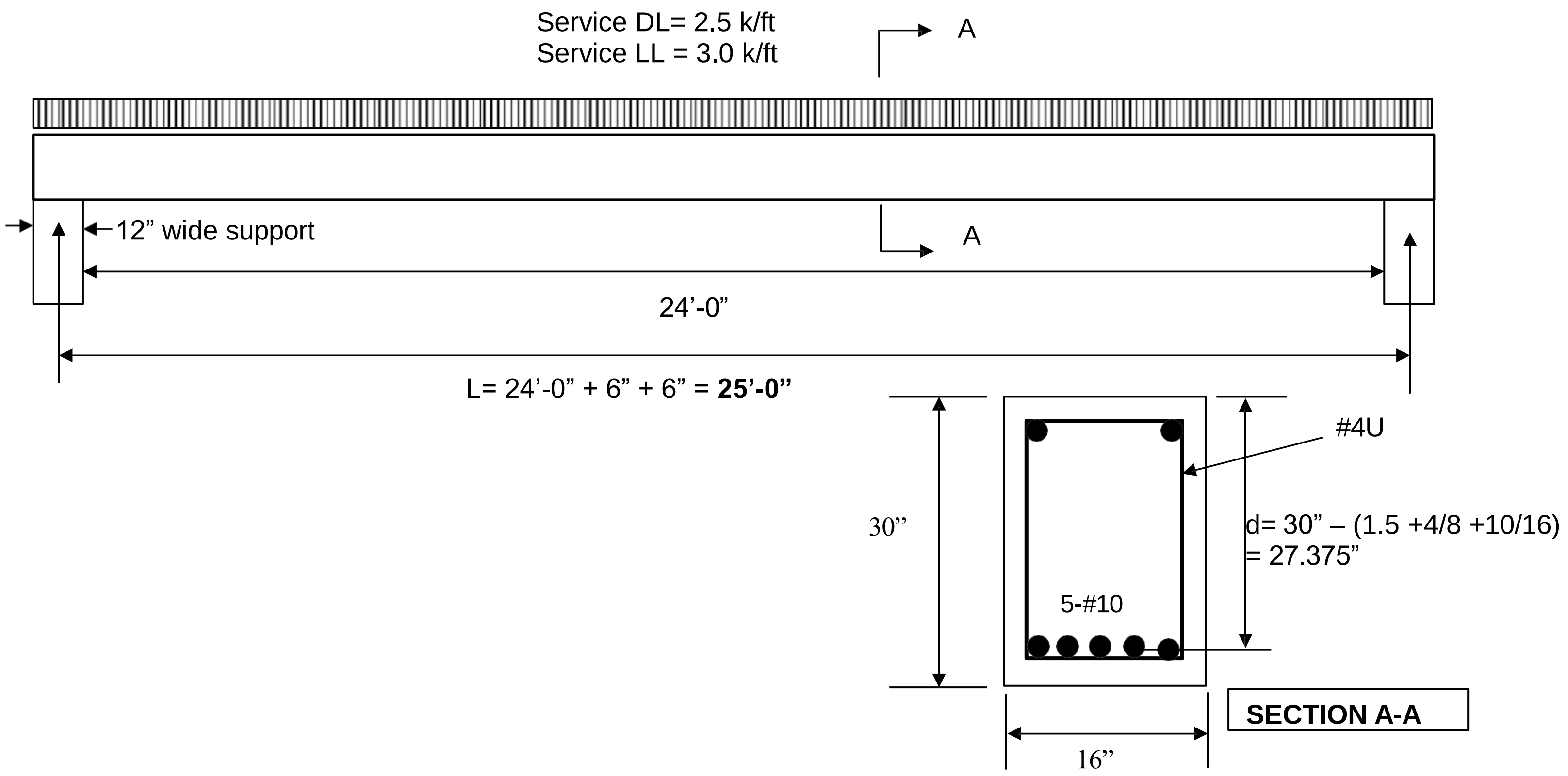
Adopted spacing : $s \leq \text{Min}\left(s_{max}^{1}, s_{max}^{2}, s_{max}^{3}\right) \Rightarrow s = 210\,mm$

$\Rightarrow$ We use a 200 mm spacing

Design of Stirrup

WEEK- 14

Q1: Design stirrup for the simply-supported beam as shown in Figure. Use #4 U stirrups. The supports are 12” wide, and the loads shown are service loads. Beam cross-section is shown in SECTION A-A.
Use f_c’=3000 psi, and f_{yt} = 40000 psi.



Solution:

INPUT		
L=	25	ft
DL=	2.5	k/ft
LL=	3	k/ft
h=	30	in
bw=	16	in
fc'=	3000	psi
fyt=	40000	psi

d	27.375	
SW=	0.5	k/ft
wu=	8.4	

Lamda	1	

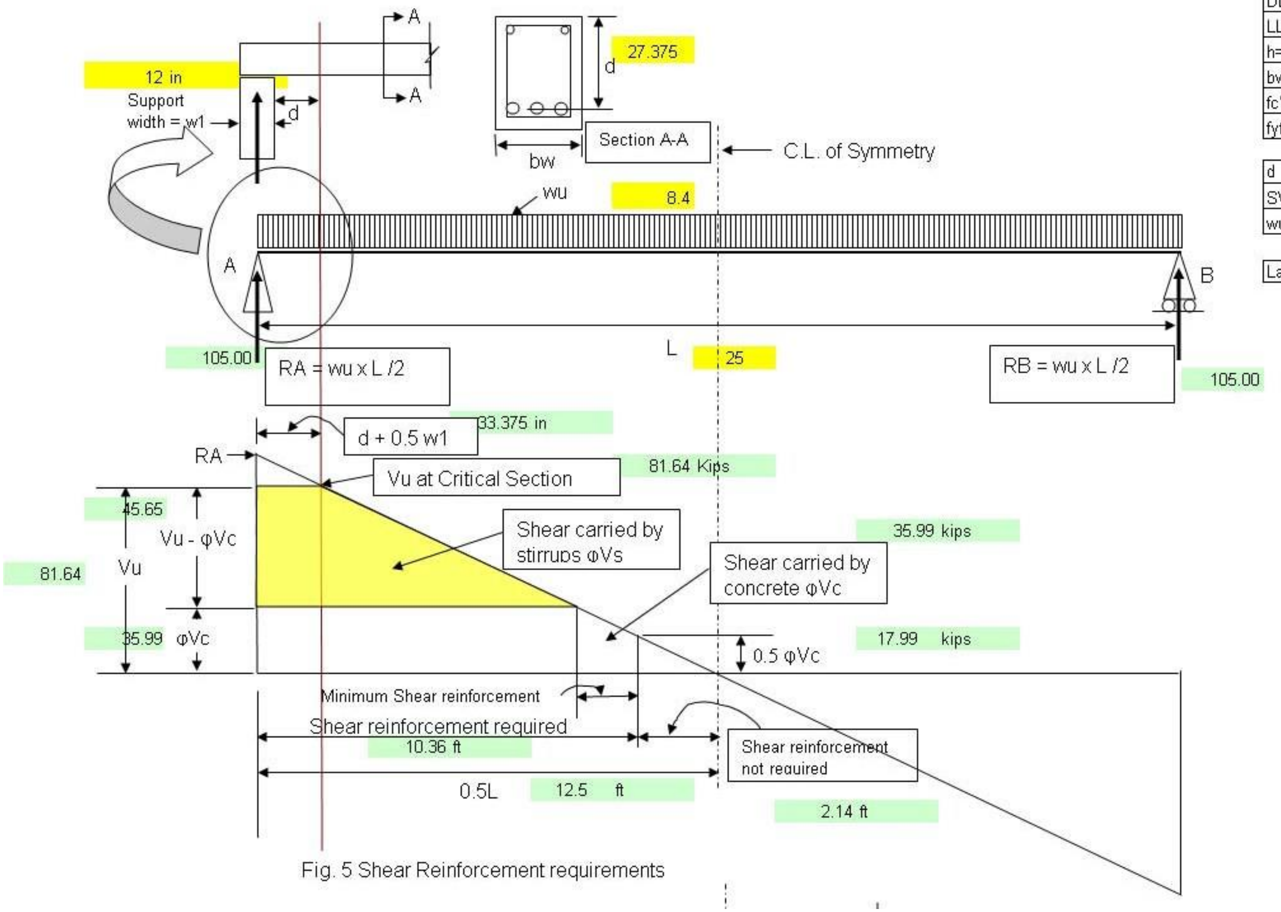


Fig. 5 Shear Reinforcement requirements

1. Draw Shear, V_u Diagram (Fig. 5)

   Bean self weight, SW= [(30x16)/144] 0.15 = **0.5 k/ft**

   Wu = 1.2(D + SW) + 1.6 (L) = 1.2(2.5+0.5) + 1.6(3.0) = **8.4 k/ft**

   Reactions, $R_A = R_B$ = Wu L /2 = 8.4 x 25 /2 = **105 kips**

2. Calculate V_u at a distance d from the face of support; Vu @d = 105 – 8.4(27.375+6)/12 = **81.64 kips**

3. On the Vu diagram, identify locations where (1) Shear Reinforcement required, (2) where shear reinforcement not required, (3) where shear carried by stirrups, φV_s , and (4) where minimum shear reinforcement required (Shear carried by concrete, φV_c). [Note: SEE Fig. 5]

4. Calculate $\varphi V_c = 2\,\lambda\varphi\sqrt{f_c'}\,(b_w \times d)$= 2(1)(0.75)√3000 x (16x27.375) /1000= **35.99 kips**

   φ **= 0.75;**

   λ **= 1** for normal weight concrete; 0.85 for sand-lightweight concrete; 0.75 for all lightweight concrete.

5. Calculate $\varphi V_s = [V_u - \varphi V_c]$ = 81.64 – 35.99 = **45.65 kips**

   <u>Check:</u> If $8\phi\sqrt{f_c'}.\ b_w\, d < [\phi V_s]$, then SECTION SHOULD BE ENLARGED [STOP AT THIS STEP]

   $8\phi\sqrt{f_c'}.\ b_w\, d < [\phi V_s]$ = 8(0.75) √3000 (16 x 27.375) /1000 = **143.94 kips > φV_s**

6. No Stirrups are needed if $V_u < 0.5\,\varphi\, V_c$ **81.64 > (0.5 x 35.99) YES, <u>STIRRUPS REQUIRED</u>**

DESIGN STIRRUPS

7. Determine required spacing of vertical U stirrups based on φV_s

Calculate theoretical stirrup spacing, $S = \varphi\ A_v \times f_y \times d / [V_u - \varphi V_c]$

Use #4 U Stirrups:

$S = 0.75\ (2x0.2)(40)(27.5)/[45.65]$ = **7.2 in (CONTROLS)**

S must satisfy

$S \le d/2 \le 24$ inch

S<= d/2=27.375/2 = 13.69 in
S<= 24 in

If $[V_u - \phi V_c] > 4\phi\sqrt{f_c'}.\ b_w\ d$ Then $S \le d/4$

$\{4\phi\sqrt{f_c'}.\ b_w\ d\}$ = 144.6 /2 = 71.97 kips > 45.65 kips. Therefore, S<=d/4 Not Controls.

USE #4 U @ 7 in

8. Determine spacing of vertical U stirrups based on minimum shear reinforcement.

S is smaller of the two:
For #4U Stirrups
$S = A_v\ f_y /[50\ b_w]$ = (2 x 0.2)(40000)/[50x 16] = 20 in

$S = A_v\ f_y /[\ 0.75\sqrt{f_c'}\ .\ b_w]$ = (2x0.2)(40000)[0.75 x √3000 x 16] = 24.34 in

S must satisfy
$S \le d/2 \le 24$ inch
S=d/2 = 27.375/2 = **13.69 in (CONTROLS)** **USE #4 U @ 13.5 in O.C.**

S=24 in

Shear reinforcement **NOT required** from the C.L. of the beam to a distance of = (0.5x36.35)/8.4 = **2.15 ft**

Shear reinforcement required from support to a distance = 12.5 - 2.15 = **10.35 ft**

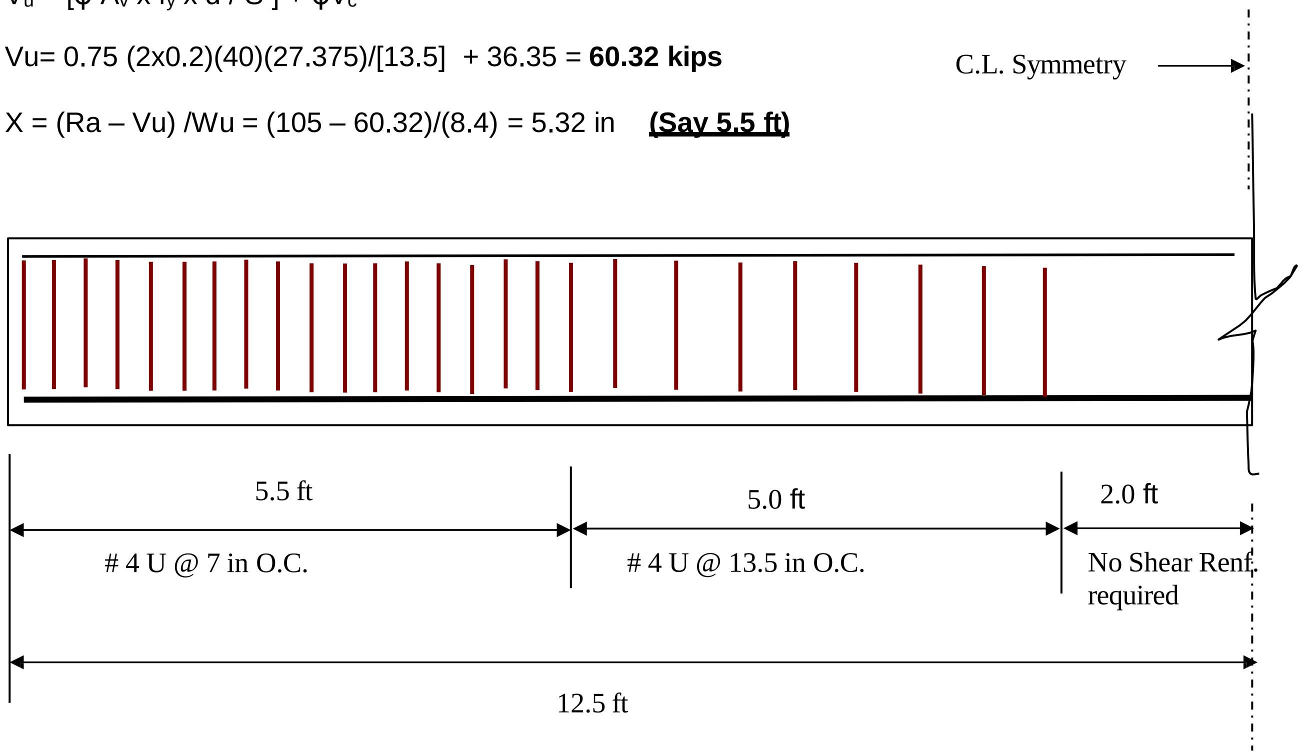
Distance "X" from the support beyond which #4 @ 13.5 in O.C. stirrup spacing can be used.

First find out for what Vu value will require #4 U @ 13.5 in

$V_u = [\varphi\ A_v \times f_y \times d / S] + \varphi V_c$

Vu= 0.75 (2x0.2)(40)(27.375)/[13.5] + 36.35 = **60.32 kips**

X = (Ra – Vu) /Wu = (105 – 60.32)/(8.4) = 5.32 in **(Say 5.5 ft)**

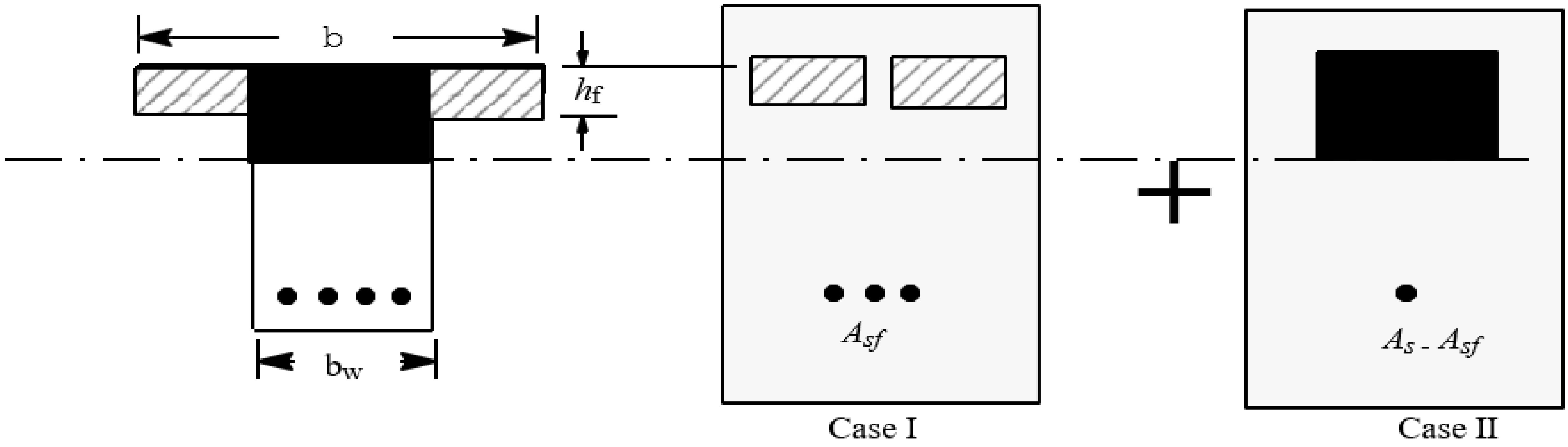


Design of T-Beam

WEEK-15

Example.- Design of T-Beams in Bending- Determination of Steel Area for a given Moment:

A floor system consists of a 3 in. concrete slab supported by continuous T beams of 24 ft span, 47 in. on centers. Web dimensions, as determined by negative-moment requirements at the supports, are b_w = 11 in. and d = 20 in. What tensile steel area is required at midspan to resist a moment of 6,400 in-kips if f_y = 60,000 psi and f'_c = 3,000 psi.



Solution

First determining the effective flange width from Section (8.3.1.) or ACI 8.10.2

1) $b \leq \frac{span}{4} = \frac{24 \times 12}{4} = 72 \; in$

2) $b \leq 16h_f + b_w = (16 \times 3) + 11 = 59 \; in$

3) $b \leq$ clear spacing between beams $+ \; b_w =$ center to center spacing between beams = 47 in

The centerline T beam spacing controls in this case, and b = 47 inches.

Assumption: Assuming that stress-block depth equals to the flange thickness of 3 inches (beam behaves like a rectangular shape).

$$A_s = \frac{M_u}{\varphi f_y(d - a/2)} = \frac{6400}{0.9 \times 60 \times (20 - 3/2)} = 6.40 \; in^2$$

Solve for "a":

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6.40 \times 60}{0.85 \times 3 \times 47} = 3.2 \ in > h_f = 3.0 \quad \text{Assumption incorrect}$$

Therefore, the beam will act as a T-beam and must be designed as a T-beam. From Case I given above and Section (8.4.1.) we have

$$A_{sf} = \frac{0.85 f_c' \, h_f (b - b_w)}{f_y} = \frac{0.85 \times (3ksi) \times (3in) \times (47 - 11)}{60(ksi)} = 4.58 \ in^2$$

$$\varphi M_{n1} = \varphi A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 0.9 \times 4.58 \times (60ksi) \times (20 - 3/2) = 4570 \ \text{in-kips}$$

$$\varphi M_{n2} = M_u - \varphi M_{n1} = 6400 - 4570 = 1830 \ \text{in-kips}$$

Find "a" value by iteration. Assume initial a = 3.5 inches

$$A_s - A_{sf} = \frac{\varphi M_{n2}}{\varphi f_y(d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 3.5/2)} = 1.86 \ in^2$$

Find an improve "a" value

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' \, b_w} = \frac{1.86 \times 60}{0.85 \times 3 \times 11} = 3.97 \ in$$

Iterate with the new a = 3.97 in.

$$A_s - A_{sf} = \frac{\varphi M_{n2}}{\varphi f_y(d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 3.97/2)} = 1.88 \ in^2$$

Find an improve "a" value

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' \, b_w} = \frac{1.88 \times 60}{0.85 \times 3 \times 11} = 4.02 \ in$$

$$A_s - A_{sf} = \frac{\varphi M_{n2}}{\varphi f_y(d - a/2)} = \frac{1830}{0.9 \times 60 \times (20 - 4.02/2)} = 1.88 \ in^2$$

Since there is no change between equations (8.13) and (8.15) we have arrived at the answer. Therefore,

$$A_s = A_{sf} + (A_s - A_{sf}) = 4.58 + 1.88 = 6.46 \ \ in^2$$

Check with ACI requirements for maximum amount of steel (Tension-Controlled)

$$c = \frac{a}{\beta_1} = \frac{4.02}{0.85} = 4.73$$

$$\frac{c}{d} = \frac{4.73}{20} = .237 < 0.375 \qquad \textit{Tension-controlled}$$

Therefore, the T-beam satisfies the ACI provisions for tension failure. Next steps will be to select the reinforcement and check all the spacing requirements and detail the beam.


THANK YOU!